1/09/18

# Digital Logic and Design:

(i) Number system and signed data representation (VIMP)

(ii) Boolean algebra and logic gates

(iii) k-maps

(iv) Combinational circuits

(v) Sequential circuits (VVVIMP)

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## 1. Boolean Algebra:

→ It is operated with binary variables.

→ Let x, y, and z are different binary variables, different theorems and postulates are:

* $x+0=x$
* $x+1=1$
* $x \cdot 1 = x$
* $x \cdot 0 = 0$
* $x+x = x(1+1) = x$
* $x+\bar{x} = 1$
* $x \cdot \bar{x} = 0$
* $x+xy = x(1+y) = x$

distributive law

$x+yz = (x+y)(x+z)$

** $(x+y)\cdot(x+z) = x+xz+xy+yz$

$= x(1+z+y)+yz$

$= x+yz$ (distributive law)

* $x+\bar{x}y = (x+\bar{x})(x+y) = x+y$
* $\bar{x}+xy = (\bar{x}+x)(\bar{x}+y) = \bar{x}+y$
* $\bar{x}+\bar{x}y = \bar{x}(1+y) = \bar{x}$

## Demorgan's laws:

(i) $\overline{A+B} = \bar{A} \cdot \bar{B}$

$\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$

(ii) $\overline{A \cdot B} = \bar{A} + \bar{B}$

$\overline{A \cdot B \cdot C} = \bar{A} + \bar{B} + \bar{C}$

## Logic Gates:

→ It is an electronic circuit, delivers only one output by permitting one or more inputs.

→ Except NOT gates all gates are designed with minimum two inputs.

★ → AND gate, OR gate, NAND gate, NOR gates can have two or more inputs and ~~thge~~ these gates are known as single input signal controlled gates. Hence they are not suitable for staircase operation.

★ → Generally x-OR gate and x-NOR gates design is available with only two inputs because multiple input x-OR gate / x-NOR gates design cost is too expensive and these gates are perfectly suitable for staircase operation.

Q: A lamp (L.E.D) is operated with 2 switches. First switch is located at first floor and 2nd switch at second floor. Any one of these two switches can be used to control lamp irrespective of other switch position. This logic implies

a) AND b) OR c) NAND d) EXOR ✓

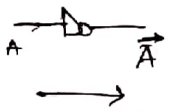
# Logic gates

- Basic: OR, AND, NOR
- Universal: NAND, NOR
- Derived: Ex-OR, Ex-NOR

**Universal gates:**

The gate from which any expression circuit can be achived.

## NOT gate (Inverter)

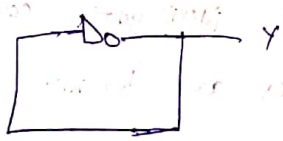



$t_{pd}$ (Propagation delay).

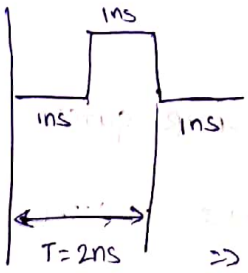
↓

Time taken by a gate to produce o/p from I/p.

**Q:** In the below logic diagram, $t_{pd}$ of NOT gate is 1ns. what is the signal frequency at Y?




a) 1GHz  b) 2GHz  c) 0.5GHz ✓  d) None




$\Rightarrow f = 0.5\,GHz$

## AND gate:


$Y = A \cdot B = A \cap B$

## NAND gate:


$Y = \overline{A \cdot B}$

## OR gate:


$Y = A + B = A \cup B$

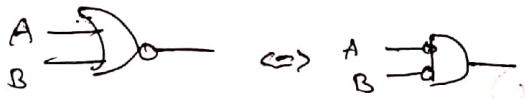
## NOR gate


$Y = \overline{A+B}$

By De morgan's 1st law

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

* NOR gate = bubbled AND gate.



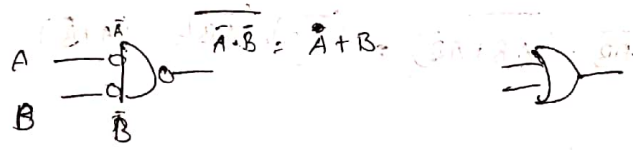

By De morgan's 2nd law

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

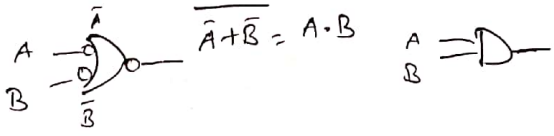
* NAND gate = bubbled OR gate


Similarly

* Bubbled NAND gate ~ to OR gate




* Bubble NOR gate ~ to AND gate



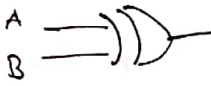

* Any of AND, OR, NAND, NOR gates o/p can be sometimes decided with single input ~~then~~. Hence they are known as Single input signal controlled gates which makes unusefull for staircase operation

* Staircase operation: It is process of controlling a device by either of two Switches irrespective of position (state) of other switch.

**Ex OR gate**: o/p is high when it's input is having odd no of 1's.

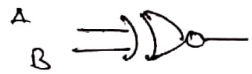
**EXNOR**: o/p is high when its i/p is having even no of 1's.

**EXOR** (Non-coincidence gate)



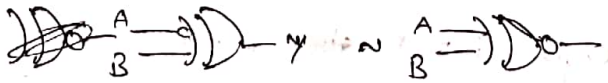

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**Ex-NOR** (Coincidence gate)

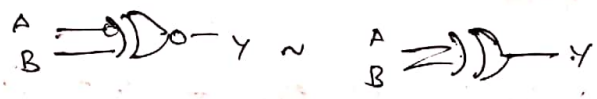



| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

* Single bubbled EXOR ~ to Ex-NOR




* Sngle bubbled Ex-NOR ~ to Ex-OR




* Bubble EXOR is EXOR itself (So is Ex-NOR)

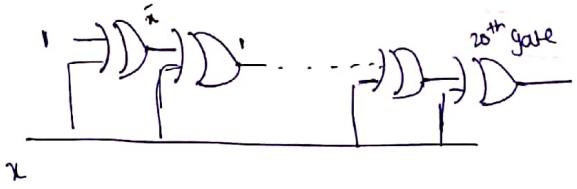
→ $x \oplus x = 0$ $\quad x \oplus \bar{x} = 1$

→ $x \odot x = 1$ $\quad x \odot \bar{x} = 0$

→ $x \oplus 0 = x$ $\quad x \oplus 1 = \bar{x}$

→ $x \odot 0 = \bar{x}$ $\quad x \odot 1 = x$

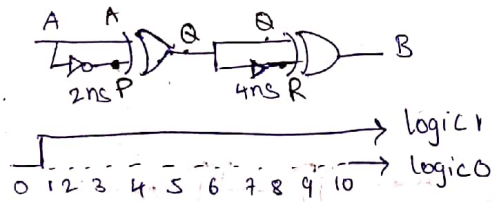
Q: In the below diagram, what is the o/p at 20th gate?




a) $x$ b) $\bar{x}$ c) 1 ✓ d) 0

Q: In the below diagram, there is no $t_{pd}$ for XOR gates and all gates initial i/p & o/p are 0. First non-inverting buffer takes 2ns and second buffer takes 4ns.

non-inverting buffer → o/p is same as i/p except it provides some delay.

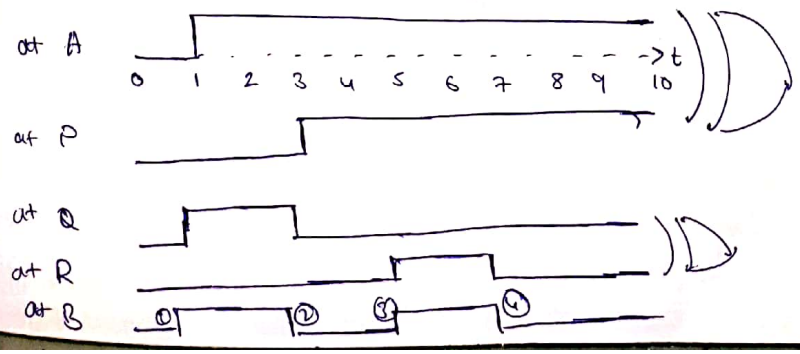



The above is applied at 'A'. The no of transitions (changes) occured at B from 0 to 10 ns time is _____

a) 1 b) 2 c) 3 d) 4 ✓

Sol:

Method 1:

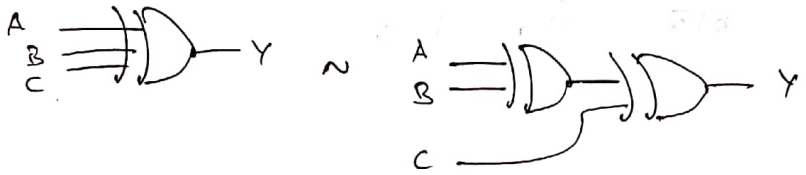



⇒ (4)

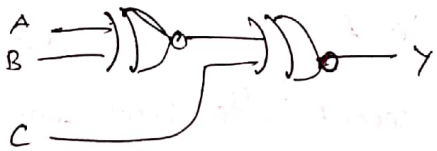
Method 2 :

| time | A | P | Q | R | B | |
|---|---|---|---|---|---|---|
| 0-1 | 0 | 0 | 0 | 0 | 0 | ① |
| 1-2 | 1 | 0 | 1 | 0 | 1 | |
| 2-3 | 1 | 0 | 1 | 0 | 1 | ② |
| 3-4 | 1 | 1 | 0 | 0 | 0 | |
| 4-5 | 1 | 1 | 0 | 0 | 0 | ③ |
| 5-6 | 1 | 1 | 0 | 1 | 1 | |
| 6-7 | 1 | 1 | 0 | 1 | 1 | ④ |
| 7-8 | 1 | 1 | 0 | 0 | 0 | |
| 8-9 | 1 | 1 | 0 | 0 | 0 | |
| 9-10 | 1 | 1 | 0 | 0 | 0 | |

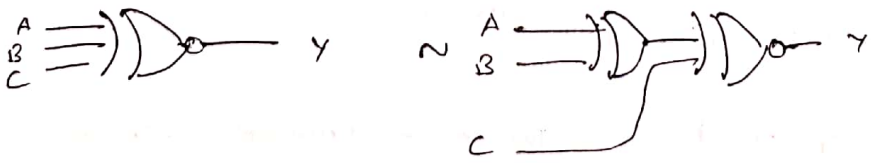
3-input Ex-OR



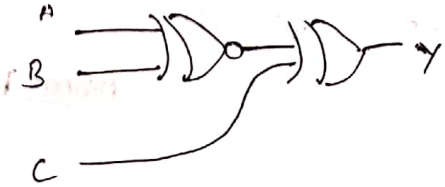

(or)




3-input Ex-NOR




(or)



## Exclusive gates special properties:

Let $A \oplus B = Y$

$\Rightarrow A \oplus B = 0 \oplus Y$

$A \oplus B \oplus Y = 0$

$\Rightarrow A = B \oplus Y$

$B = A \oplus Y$

$\bar{A} \oplus \bar{B} = Y$

Let $A \oplus B \oplus C = Y$

$A \oplus B = Y \oplus C$

$A \oplus B \oplus C \oplus Y = 0$

$\bar{A} \oplus \bar{B} = Y \oplus C$

$A \oplus B = \bar{Y} \oplus \bar{C}$

$\bar{A} \oplus B = \bar{Y} \oplus C$

$\bar{A} \oplus B = Y \oplus \bar{C}$

(A-variable can be shifted from L.H.S to R.H.S (or) R.H.S to L.H.S along with operator. This is a special property of exclusive gates)

## Problems

5) A) $\bar{A} + \bar{B} = \overline{A \cdot B}$

B) $\overline{A+B} = \bar{A} \cdot \bar{B}$

C) $\overline{\bar{A}+\bar{B}} = AB$

D) $\overline{\bar{A} \cdot \bar{B}} = A + B$

6) $A \oplus B = 0 \iff A = B$

$\overline{A+B} = 0 \iff A = 1$ (or) $B = 1$

$\bar{A} \cdot B = 0 \iff A = 1$ (or) $B = 0$

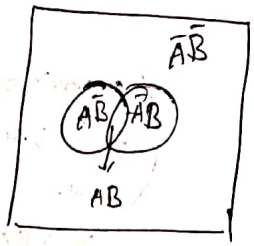
$A \oplus B = 1 \iff A \neq B$

## Venn diagrams for logic gates:

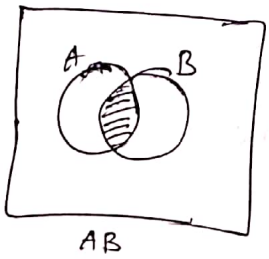
→ venn diagram is collection of circles.

→ one circle is required for one input.

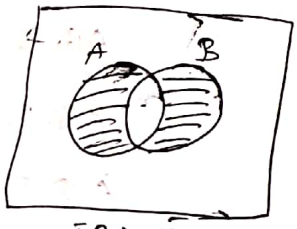
→ In venn diagram shaded area represents High output.



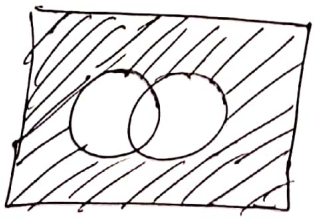

Venn diagram for AND gate:

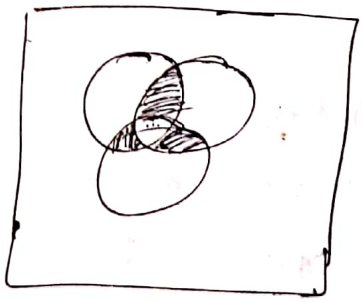



Ex-OR




NOR




## Different methods used to obtain expression from given truth table:

(i) Apply S.O.P (minterm)

(ii) Apply P.O.S (maxterm)

(iii) Apply k-map

(iv) Apply tabulation method

Note: Result is same from any method

→ k-map technique is used when given function is having don't cares(x).

→ k-map range is 2 to 5 variables

→ when the given function is having more than 5 variables, apply tabulation technique.

→

| | A B | Y | |
|---|---|---|---|
| 0 | 00 | 0 | ~~minterm~~ maxterm |
| 1 | 01 | 0 | ~~maxterm~~ maxterm |
| 2 | 10 | 1 | ~~maxterm~~ minterm |
| 3 | 11 | 1 | ~~maxterm~~ minterm |

For above table

(i) SOP: (minterm)

$= m_2 + m_3$

$A\bar{B} + AB$

$= A(B+\bar{B}) = A(1) = A$

(ii) POS (maxterm)

$M_0 \cdot M_1$

$= (A+B)(A+\bar{B})$

$= A + A\bar{B} + AB$

$= A(1+B+\bar{B})$

$= A(1)$

$= A$

| | I/P x y | Term Designation Σ | S.O.P '1' | Π | P.O.S '0' |
|---|---|---|---|---|---|
| 0 | 0 0 | $\bar{x}\bar{y}$ | $m_0$ | $x+y$ | $M_0$ |
| 1 | 0 1 | $\bar{x}y$ | $m_1$ | $x+\bar{y}$ | $M_1$ |
| 2 | 1 0 | $x\bar{y}$ | $m_2$ | $\bar{x}+y$ | $M_2$ |
| 3 | 1 1 | $xy$ | $m_3$ | $\bar{x}+\bar{y}$ | $M_3$ |

→ Sum of all minterms is high (1)

$$m_0 + m_1 + m_2 + m_3$$

$$= \bar{x}\bar{y} + \bar{x}y + x\bar{y} + xy$$

$$= \bar{x}(y+\bar{y}) + x(y+\bar{y})$$

$$= x + \bar{x}$$

$$= 1$$

→ Product of all maxterms is low (0)

$$M_0 \cdot M_1 \cdot M_2 \cdot M_3$$

$$\frac{(x+y)(x+\bar{y})(\bar{x}+y)(\bar{x}+\bar{y})}{(x+y\bar{y})(\bar{x}+y\bar{y})}$$

$$(x+0)(\bar{x}+0)$$

$$(x\bar{x}) = 0$$

## Canonical form :

→ It is also known as standard form

→ An SOP/POS expression is said to be in canonical form, when given terms are represented as functional variables only.

→ Cannonical form is used to prepare the truth table for the given minimized expression

Ex: Prepare truth table for

$$f(ABC) = A\bar{B} + B\bar{C}$$

$f(ABC) = A\bar{B} + B\bar{C}$

$A\bar{B}(C+\bar{C}) + (A+\bar{A})B\bar{C}$

$A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$

101 100 110 010

$m_5 + m_4 + m_6 + m_2$

$\sum_m(2,4,5,6) = \prod M(0,1,3,7)$

| ABC | Y |
|---|---|
| 000 | 0 |
| 001 | 0 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 1 |
| 110 | 1 |
| 111 | 0 |

Shortcut :

| | ABC | |
|---|---|---|
| $A\bar{B}$ | 10X | |
| | 100 | (4) |
| | 101 | (5) |
| $B\bar{C}$ | X10 | |
| | 110 | (6) |
| | 010 | (2) |

$\Rightarrow \sum_m(2,4,5,6)$

Eg: 2 $f(ABC) = A + \bar{B}C$

| | ABC | |
|---|---|---|
| A | 1XX : | (4,5,6,7) |
| $\bar{B}C$ | X01 : | (5,1) |

$\Rightarrow \sum m(1,4,5,6,7)$

## Redundant term:

→ It is a repeated term.

→ It's removal from expression does not disturb it's original form

Eg:3 Find redundant term in

$f(ABC) = A\bar{B} + \bar{A}C + \bar{B}C$

a) $A\bar{B}$ b) $\bar{A}C$ c) $\bar{B}C$ ✓ d) None

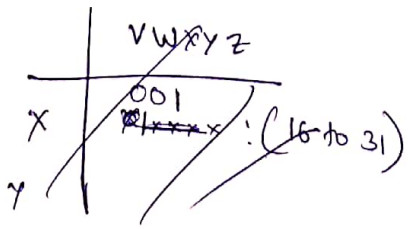
| | ABC |
|---|---|
| $A\bar{B}$ | 10x : (4,5) |
| $\bar{A}C$ | 0x1 : (1,3) |
| $\bar{B}C$ | x01 : (1,5) |

$\Sigma_m(1,3,4,5)$

If we remove $\bar{B}C$ still $m_1$ & $m_5$ exists. So $\bar{B}C$ is the redundant terms

## Problems

(2)

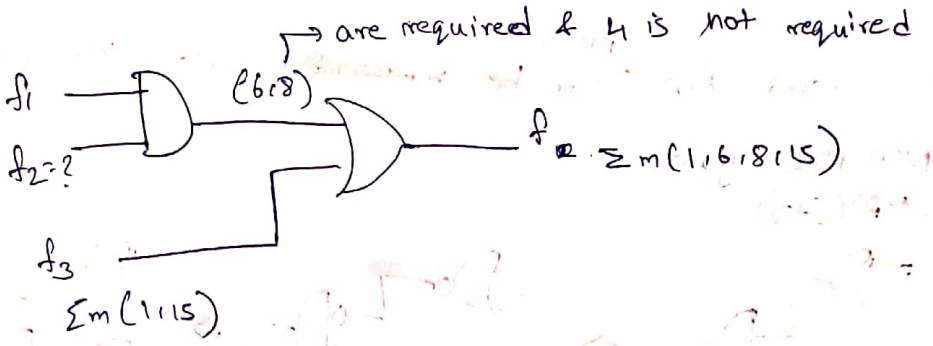



| | VWXYZ | |
|---|---|---|
| x | xx1xx → 16 | common: xx101 ⇒ 4 |
| $\bar{y}z$ | xxx01 → 8 | ↓ redundant terms |

∴ 16 + 8 − 4 = 20 (minterms)

no of maxterms = 32 − 20 = 12

③




2007

Q: Max no of distinct boolean functions that can be implemented with 'n' binary variables is

a) $2^n$ b) $2^{2^n}$ ✓ c) $2^{2^{n-1}}$ d) None

Consider

| | xy | $F_0$ | $F_1$ | $F_2$ | - - - - | $F_{15}$ |
|---|---|---|---|---|---|---|
| 0 | 00 | 0 | 0 | 0 | | 1 |
| 1 | 01 | 0 | 0 | 0 | | 1 |
| 2 | 10 | 0 | 0 | 1 | | 1 |
| 3 | 11 | 0 | 1 | 0 | | 1 |

$2^n$

0 0 0 0

with 2 variables, we can generate $2^2$ combinations

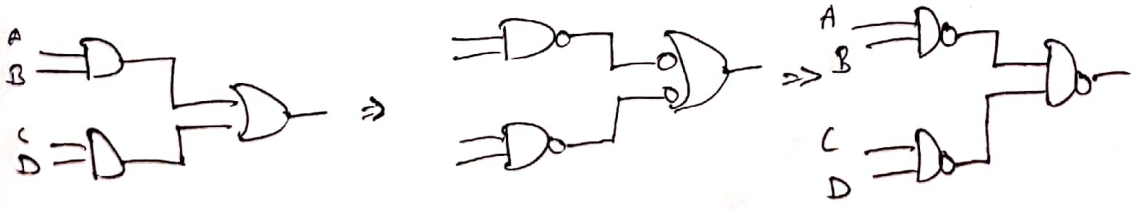
with $2^2$ combinations, max no of distinct functions $= 2^{2^2} = 16$

with n variables, we can generate $2^n$ combinations

with $2^n$ combinations, max no of distinct functions $= 2^{2^n}$

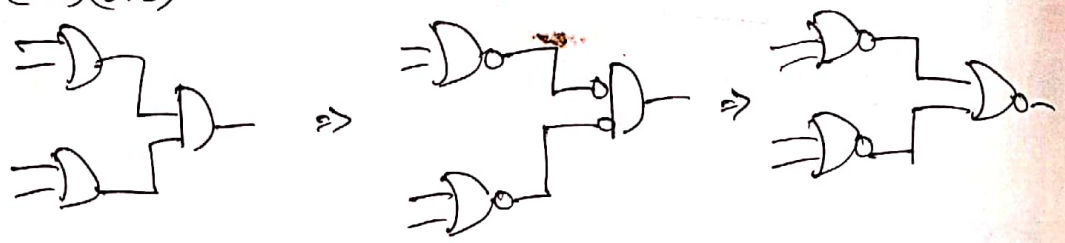
→ Two level SOP expression can be implemented with 2level NAND-NAND gates

AB+CD



→ To Two level POS expression can be ~~represented~~ implemented with 2 level NOR-NOR gates.

$(A+B)(C+D)$



## Dual function ($F^D$):

In the given function if '+' is replaced by '.' and '.' is replaced by '+' one new function will be generated.

→ This new function is dual function for the given input fn.

→ If all minterms/Maxterms produced by the input fn and its dual function are same then input function is known as self dual function (SDF)

→ Maximum no of self dual functions that can be implemented with n binary variables = $2^{2^{n-1}}$

Note:

→ To become SDF, in the given function the no of minterms must be equal to no of maxterms (It doesn't mean that a function is SDF if no of minterms & no of maxterms are equal. Its just a pre-requirement)

Eg: $F(ABCD) = AB+CD$ (S.O.P)

$F^D(ABCD) = (A+B).(C+D)$ (P.O.S)

Eg 1: Check if $F(ABC) = AB + BC + AC$ is SDF

$F(ABC) = AB + BC + AC$

| | ABC | |
|---|---|---|
| AB | 11X | (6,7) |
| BC | X11 | (3,7) |
| AC | 1X1 | (5,7) |

$\Sigma m(3,5,6,7) = \Pi M(0,1,2,4)$

no of minterms = no of maxterms

$F^D(ABC) = (A+B)(B+C)(A+C)$

| | ABC | |
|---|---|---|
| A+B | 00X | ($M_0$, $M_1$) |
| B+C | X00 | ($M_0$, $M_4$) |
| A+C | 0X0 | ($M_0$, $M_2$) |

$\Pi M(0,1,2,4) = \Sigma m(3,5,6,7)$

equal

$\therefore$ SDF

Eg 2: $F(ABC) = \bar{A}B + BC + AC$ is SDF or not?

| | ABC | |
|---|---|---|
| $\bar{A}B$ | 01X | (2,3) |
| BC | X11 | (3,7) |
| AC | 1X1 | (5,7) |

$\Rightarrow \Sigma m(2,3,5,7) = \Pi M(0,1,4,7)$

no of minterms = no of maxterms

$F^D(ABC) = (\bar{A}+B)(B+C)(A+C)$

| | ABC | |
|---|---|---|
| $\bar{A}+B$ | 10X | (4,5) |
| B+C | X00 | (0,4) |
| A+C | 0X0 | (0,2) |

~~$\Pi M(0,4,5,7)$~~

$\Pi m(0,4,5,2) = \Sigma m(1,3,6,7)$

not equal

$\therefore$ Not SDF

Eg3: $F(ABC) = A + \bar{B}C$ is SDF or not

| | ABC |
|---|---|
| A | 1xx (4,5,6,7) |
| $\bar{B}C$ | x01 (1,5) |

$\Rightarrow \Sigma_m(1,4,5,6,7) = \Pi M(0,2,3)$

no of minterms $\neq$ no of maxterms

$\therefore$ not SDF

Consider

| | x y | $F_0$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ (EXOR) | $F_7$ | $F_8$ | $F_9$ | $F_{10}$ | $F_{11}$ | $F_{12}$ | $F_{13}$ | $F_{14}$ | $F_{15}$ (Ex-NOR) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2 | 1 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 3 | 1 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| | | x | x | x | ✓ | x | ✓ | ✗ | x | x | ✗ | ✓ | x | ✓ | x | x | x |

→ The rounded ones contain equal no of minterms & maxterms.

→ Here no of variables $n = 2$.

→ $\therefore$ no of SDFs possible $= 2^{2^{n-1}} = 2^{2^1} = 4$

→ Of above 6 rounded ones 4 are SDF. The ExOR & Ex-NOR are not SDF. Rest are SDF.

Note:

In 3 variable, it is not necessary that ExOR & Ex-NOR functions are not SDF.

→ The Rounded & ticked (✓) ones are SDFs.

## Universal gate applications: ~~Any target~~

→ Any expression can be implemented by universal gates.

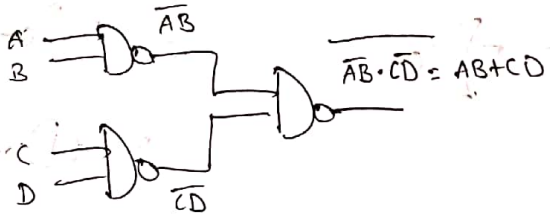
→ while implementing expression with NAND gates, first bring entire expression in SOP form then apply Demorgan's law.

→ while implementing expression with NOR gates, first bring entire expression in POS form then apply demorgan's law.

Consider

$f(A,B,C,D) = AB + CD$ (SOP form)

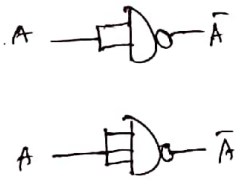
$AB+CD = \overline{(\overline{AB+CD})} = \overline{\overline{AB} \cdot \overline{CD}}$



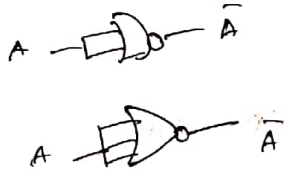

### NOT gate with universal gates:

I/P: A target $\overline{A}$

with NAND




with NOR



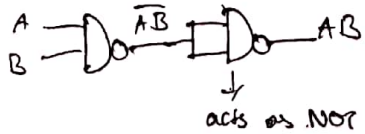

### AND gate with universal gates:

I/P: A, B target : AB available : $\overline{A \cdot B}$

with NAND

$\overline{(\overline{A \cdot B})} = AB$

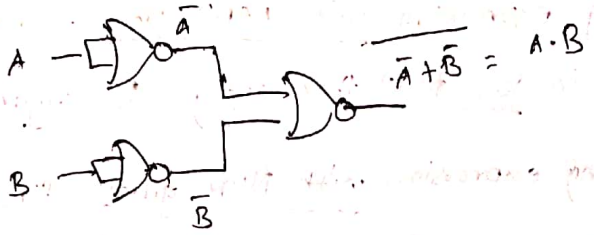
⇒



with NOR

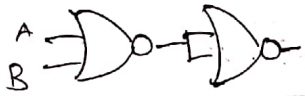
available: $\overline{A+B}$

$\bar{A}$,

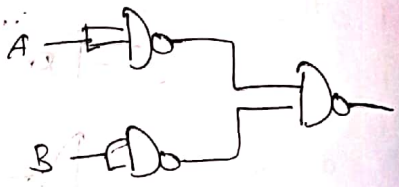



## OR gate with Universal gates

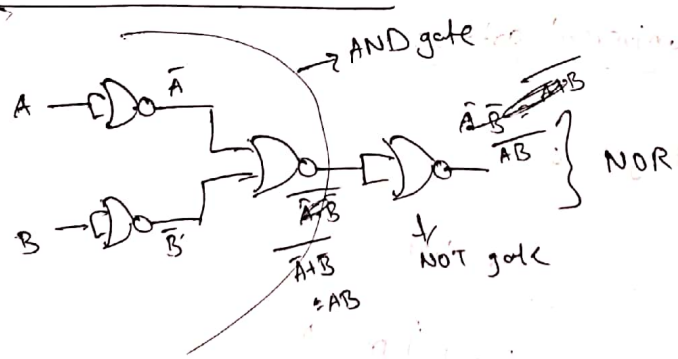
NOR with ~~NAND~~ gate



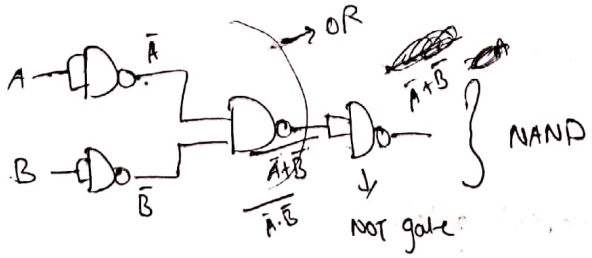

NAND with ~~NOR~~ gate




## NAND gate with NOR gate:




## NOR gate with NAND gate




$A+B$

$\overline{AB}$

$\bar{A}+\bar{B}$

Previous EC

Q: Universal gate is used to implement any target expression and we are permitted to use the required no of gates given below gate

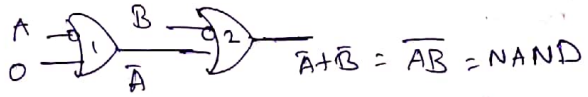
(i)  (ii)  (iii) 

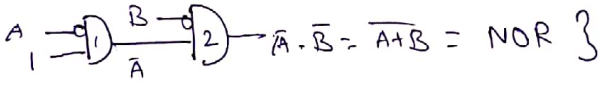
which one of the above is universal

a) ~~i~~ i b) ii c) iii d) None of the above

* It is clear that AND & OR can't be used

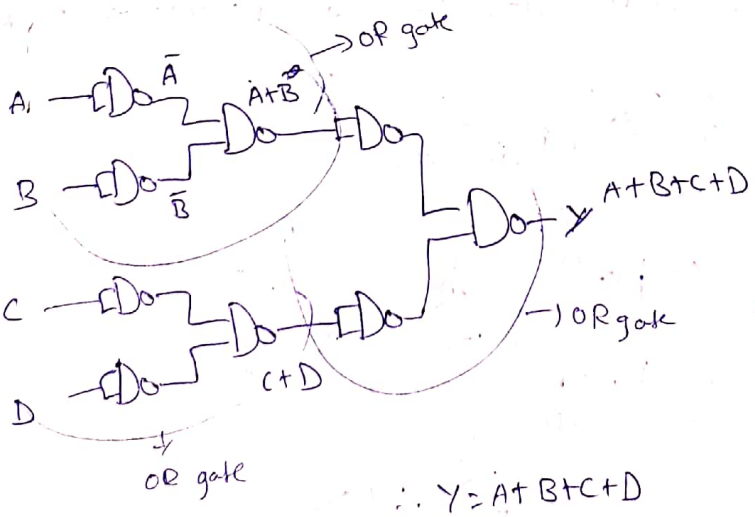
Consider




} This is just shown for example to show this is also possible.

Q: what is expression at Y




$\therefore Y = A+B+C+D$

Q: Min no of 2 i/p NAND gates to implement

$A + A\bar{B} + \bar{A}\bar{B}C + ABC\bar{D}$ is ——

$\Rightarrow A(1 + \bar{B} + \bar{B}C + BC\bar{D}) = A(1)$

$= A$

$\therefore$ no gates are required

Ans: 0

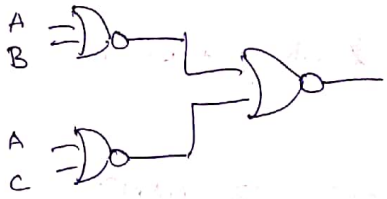
Q: Min no of 2 i/p NOR gates required to implement $(A+BC)$ is _____

$A+BC$ ↓ convert into POS

$= (A+B)(A+C)$

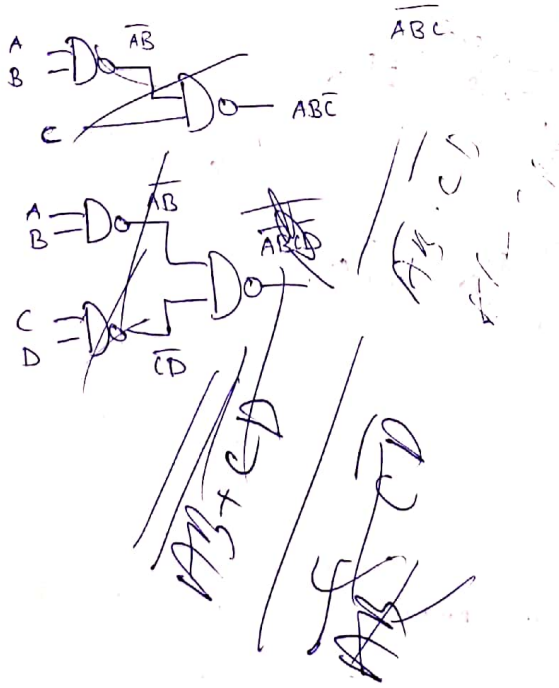
$= \overline{\overline{(A+B)(A+C)}}$

$= \overline{\overline{(A+B)} + \overline{(A+C)}}$




∴ Ans : 3

Q: Min no of 2 i/p NAND gates req to implement 4 i/p NAND gate is



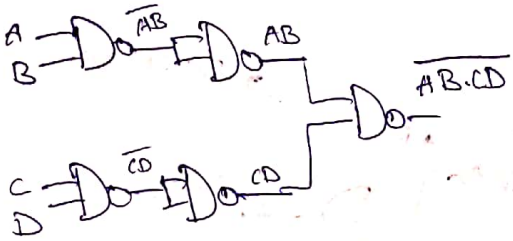

Target : $\overline{ABCD}$

Method 1 :

1st : $\overline{AB}$ 2nd : $\overline{AB} \rightarrow AB$

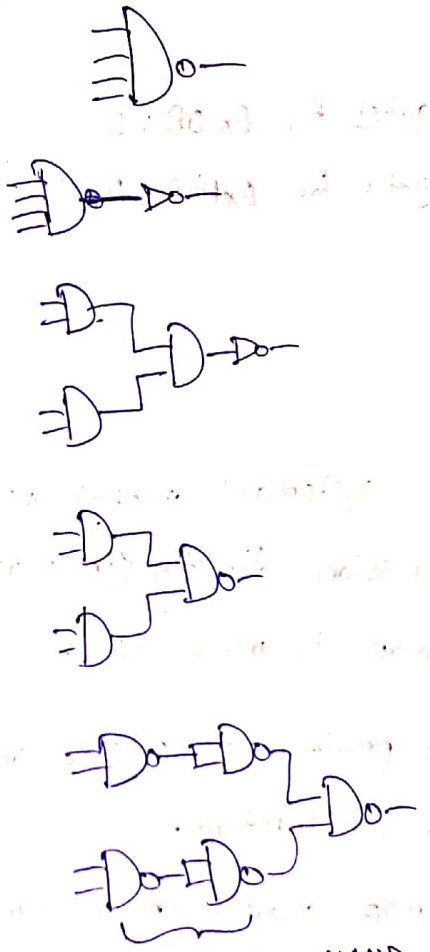
3rd : $\overline{CD}$ 4th : $\overline{CD} \rightarrow CD$

5th : $AB$ & $CD \rightarrow \overline{ABCD}$



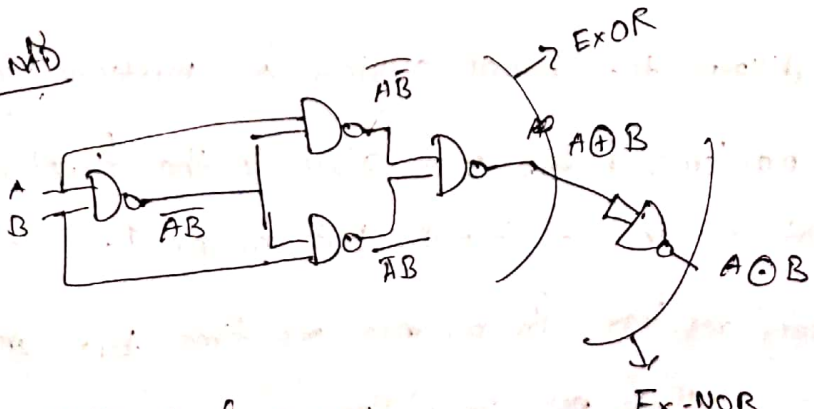

Method 2:

Target:




## Exclusive gates with NAND/NOR gates:

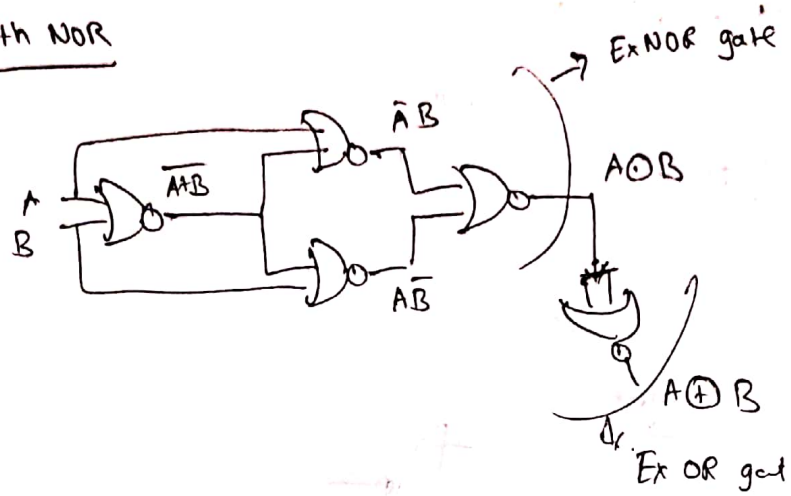
with NAND




NAND gates for EXOR: 4

NAND gates for EXNOR: 5

with NOR




No. of NOR gates for Ex OR : 5

NOR gates for ExNOR : 4

## 2. K-Maps :

It is a systematic method used to get the ~~minimum~~ minimized expression directly (in most cases). Generally one n-variable k-map is used for one n variable function.

→ n variable k-map provides $2^n$ cells, one cell is used to represent one minterm or one maxterm.

→ While using k-map services for minterms groups grouping, all the minterms, dont cares should be produced in the k-map.

→ Grouping of all min-terms is compulsary required for minimum one time but grouping of don't care cell is not compulsory

→ while utilizing the k-map services for maxterms grouping all the maxterms & dont cares given in the function should be produced in the k-map but grouping of all maxterms is compulsary required for minimum one time but grouping of don't care cell is not compulsary.

→ A cell can be grouped more than one time when its services are required.

→ The target term size will be reduced by grouping maximum no. of cells.

→ In n-variable, k-map each cell is having 'n' chances for grouping with other cells.

→ Single grouping is possible for $2^x$ cells only

Standard method used to get the minimized expression with k-map:

Step I: get prime implicants (PIs)

Step II: get essential PIs (EPIs)

Step III: check, if only EPIs are sufficient to cover all minterms or NOT

Step IV: If Yes: one more PI is not needed

If No: one (or more) PI is needed known as selective PI (SPI)

Prime Implicant: It is the minimized term obtained by grouping the max no of cells

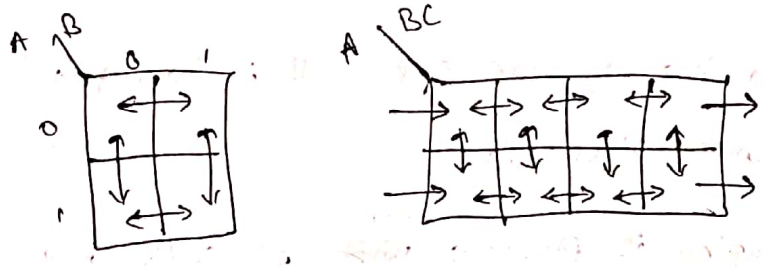
Essential PI: It's presence is essential in final minimized expression basically it is a PI.

→ A PI is said to be essential when one (or more) cell is grouped only one time

Selective PI: It is a PI, which is used to cover the minterms which are not covered by EPIs.

- Grouping: It is process of finding common bits.

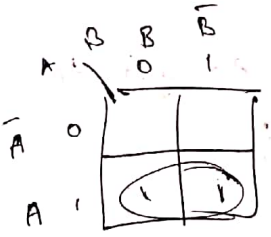
Grouping chances:




Consider

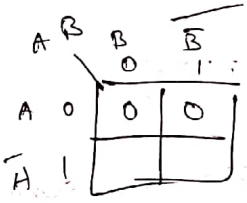
| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Using SOP




$\Rightarrow A$

POS

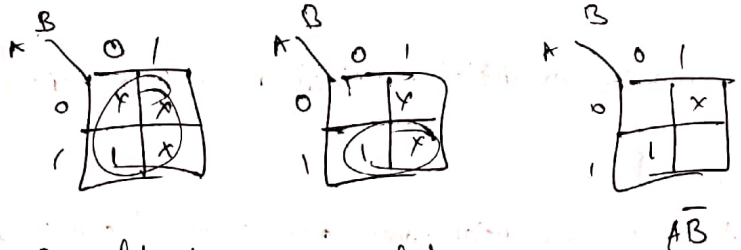



$\Rightarrow A$

Note:

→ Don't care (x) grouping is used to reduce H/W logic circuit.

Find no of useful x's and term:

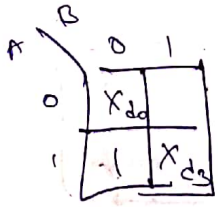



$A\bar{B}$

3 useful x's — term = '1'

1 useful — term = A

Q: Find the most useful 'x' in given below for S.O.P grouping



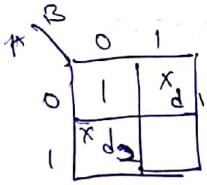

a) $d_0$ b) $d_3$ ✓ c) $d_0$ (or) $d_3$ d) none

grouping with $d_0$ : $\bar{B}$ (requires 1 NOT gate)

grouping with $d_3$ : A (requires no gates)

$\therefore d_3$

Q: Find most useful 'x' in given below for S.O.P grouping




a) $d_1$ b) $d_2$ c) $d_1$ (or) $d_2$ ✓ d) none

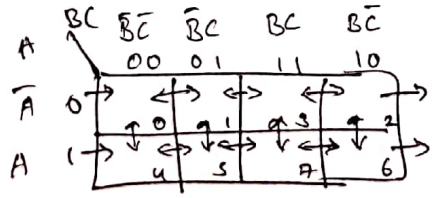
grouping with $d_1$ : $\bar{A}$ (req 1 NOT gate)

grouping with $d_2$ : $\bar{B}$ (req 1 NOT gate)

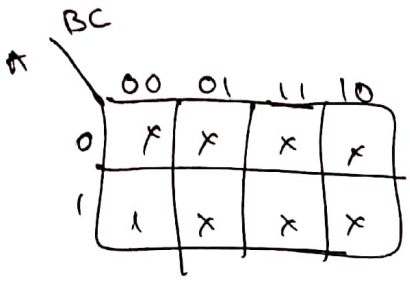
$\therefore d_1$ (or) $d_2$

Term size: No of variable available in the term.

Consider 3 variable k-map. It is drawn as below

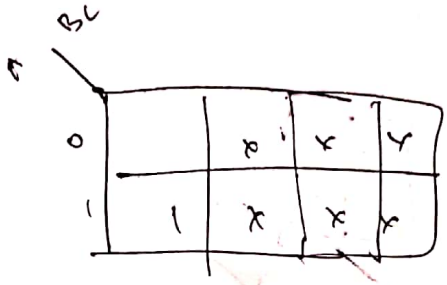


## Find term & termsize in given k-maps:



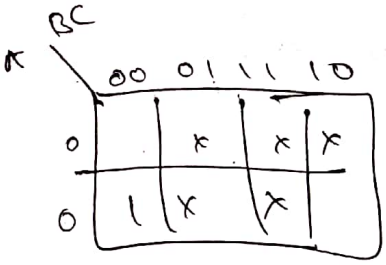

term : -

termsize : 0



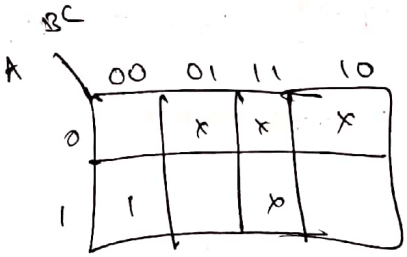

term : A

size : 1




term : $A\bar{B}$

size : 2




term $A\bar{B}\bar{C}$

size : 3

## Expression for finding the term size in 'n' variable k-map:

$$\log_2 \frac{N}{D}$$

Here N = max. no of cells given by the 0 in base 2

D = Max no of cells under single group

Get result in base 2

→ only exponent gives term size

Q: In n variable k-map, what is the term-size when 8 cells are grouped?

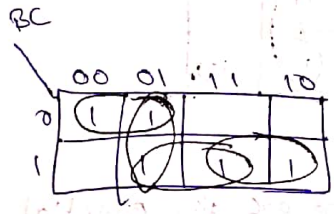
a) 3 b) 0 c) 8 d) n-3 ✓

Here $N = 2^n$

$D = 8 = 2^3$

term size $= \log_2 \frac{2^n}{2^3} = \log_2 2^{n-3} = n-3$

Q:




| Choices (PI) | EPIs |
|---|---|
| $\bar{A}\bar{B}$ | $\bar{A}\bar{B}$ |
| $\bar{B}C$ | $AB$ |
| $AC$ | |
| $AB$ | |

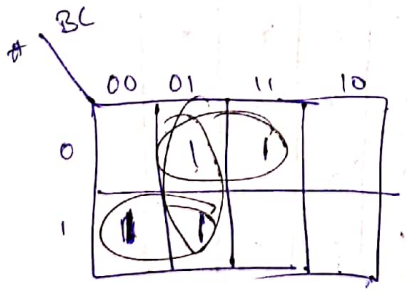
AC is selected

$f(ABC) = \bar{A}\bar{B} + AB + AC$

$\bar{A}\bar{B} + AB$ not sufficient to cover all min-terms

$AC$ ↓ SPI

Q:

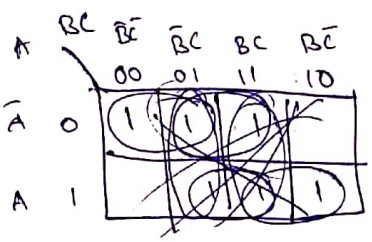


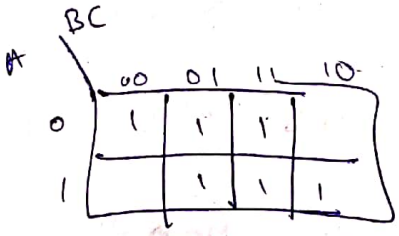

| Chances (PI) | EPIs |
|---|---|
| $A\bar{B}$ | $A\bar{B}$ |
| $\bar{B}C$ | $\bar{A}C$ |
| $\bar{A}C$ | |

$f(ABC) = A\bar{B} + \bar{A}C$

Sufficient to cover all min terms. So SPI is not needed

Q: No of EPIs in given below k-map is




a) 1 b) 2 c) 3 d) None

changes:

$\bar{A}\bar{B}$ C

$\bar{A}\bar{B}$

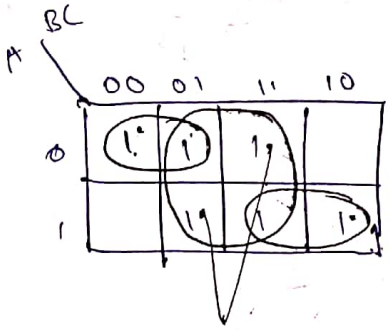
AB

EPIs

C

$\bar{A}\bar{B}$

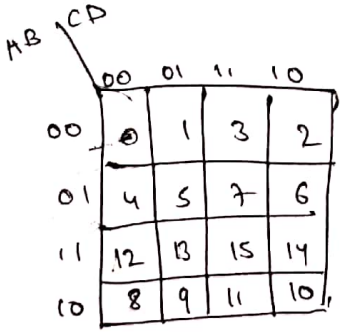
AB

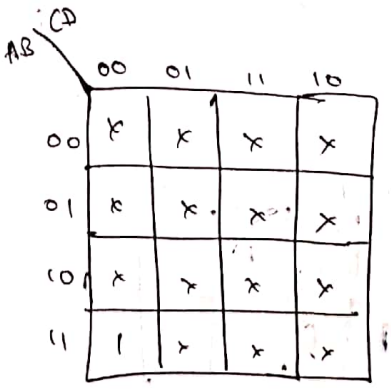



These are also essential

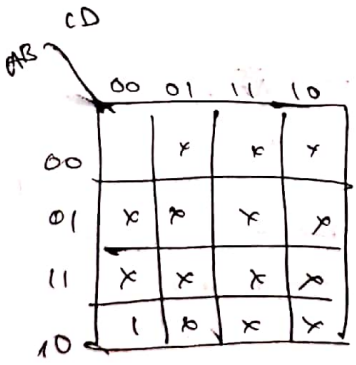
∴ C is also essential.

## 4-variable k-map:

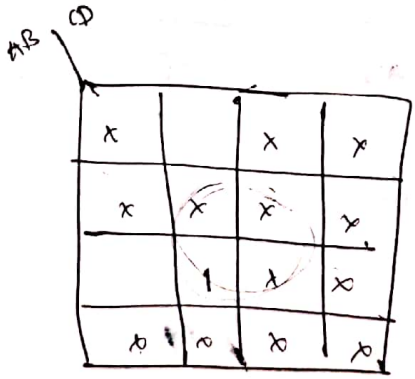


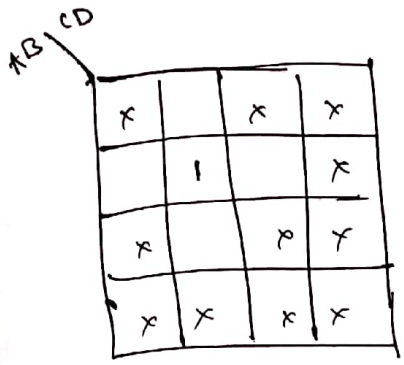



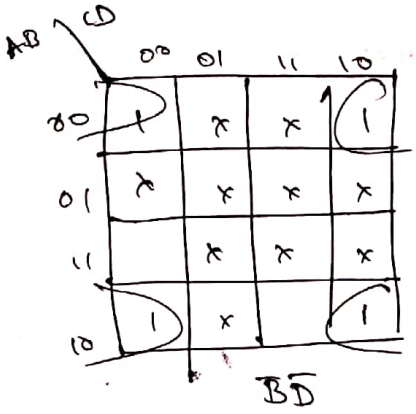

term: 1



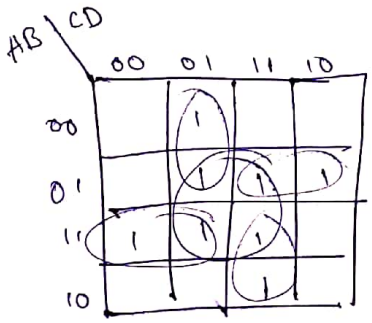

term: A






Q:- Consider below K-Map. The min no of terms appeared in final minimized expressions is _____




| Chains : | EPIs |
|---|---|
| BD | $AB\bar{C}$ |
| $AB\bar{C}$ ✓ | $\bar{A}\bar{C}D$ |
| $\bar{A}\bar{C}D$ ✓ | $\bar{A}BC$ |
| $\bar{A}BC$ ✓ | $ACD$ |
| $ACD$ ✓ | |

(Here BD is called Redundant PI)

$$f(ABCD) = AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$$

∴ Ans: 4

02/09/19

## 3. Number System

## Conversion b/w base r and $r^n$ systems:

→ Direct conversion is possible

→ Each symbol in base $r^n$ system is directly represented with 'n' bits

→ Direct conversion is possible if the largest symbol in that systems consists consists of all 1's in binary form.

Eg1: $(24)_8$ =

$(010100)_2$

Eg2: $(201A)_{16} = (0010000000011010)_2$

Eg3: $(10\ 111\ 111011011\ 101)_2 = (?)_8$

$010/111/111/011/011/101$
$2\ \ 7\ \ 7\ \ 3\ \ 3\ \ 5 = (277335)_8$

2008

Eg4: $(567)_8 = (?)_{16}$

$(567)_8 = (101\ 110\ 111)_2$

$0001/0111/0111 = (177)_{16}$
$1\ \ 7\ \ 7$

## Fraction data Conversions:

→ It requires inverse operations when compared to Integer conversions.

→ Consider $(a_{n-1} \ldots a_2\ a_1\ a_0 . b_1 b_2 \ldots b_n)_r = \sum_{i=0}^{n-1} a_i \times r^i + \sum_{i=1}^{n} b_i \times r^{-i}$

Eg: $(26.8125)_{10} = (11010.1101)_2$

$= (32.64)_8$

$= (1A.D)_{16}$

base 2

2 | 26 — 0
2 | 13 — 1
2 | 6 — 0
2 | 3 — 1
1 — 1

LIFO → 11010

~~11010~~

FIFO

.8125 × 2
1.625 × 2
1.250 × 2
0.5 × 2
1.0

→ 1101

i.e. $(11010.1101)$

In decimal form also base r to $r^n$ conversion is allowed. But after decimal zeroes are appended at end

Consider

$(11010.1101)_2 = (?)_8$

$(11010)_2 = (011010)_2 = (32)_8$

$(.1101)_2 = (.110100)_2 = (64)_8$

$(11010.1101)_2 = (32.64)_8$

base 8

$(26)_{10} = (32)_8$

.8125 × 8
6.5000 × 8
4.0

→ 64

$\Rightarrow (26.8125)_{10} = (32.64)_8$

base 16 :

$(26)_{10} = (1A)_{16}$

.8125 × 16
13.0000
D

$\Rightarrow (26.8125)_{10} = (1A.D)_{16}$

Now Consider

$(11010.1101)_2 \Rightarrow (11010)_2 = (26)_{10}$

$2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}$
.1101 $\Rightarrow 1\times\frac{1}{2} + 1\times\frac{1}{4} + 0\times\frac{1}{8} + 1\times\frac{1}{16}$

$= \frac{8+4+0+1}{16} = \frac{13}{16} = 0.8125$

$\Rightarrow 26.8125$

→ $(32.64)_8$ $(32)_8 = (26)_{10}$

64

$\frac{6}{8} + \frac{4}{64} = \frac{52}{64} = \frac{13}{16} = 0.8125$

$\Rightarrow (26.8125)_{10}$

→ $(1A.D)_{16}$

$(1A)_{16} = (26)_{10}$

$16^{-1}$

$D \Rightarrow \frac{13}{16} = 0.8125$

Eg:

Q: Let $\frac{(312)_x}{(20)_x} = (13.1)_x$ . Find x.

⇒ Convert into base 10

$$\frac{3x^2+x+2}{2x} = x+3+\frac{1}{x}$$

$$\frac{3x^2+x+2}{2x} = \frac{x^2+3x+1}{x}$$

$$3x^2+x+2-2x^2-6x-2=0$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x=5 \text{ (or) } x=0$$

## Signed data representation:

→ In unsigned binary data all the bits are used to provide magnitude (value), but in signed binary data MSB is reserved for representing sign and remaining bits are used for providing value.

→ 0 is occupied in MSB position for representing +ve data. otherwise it is 1

→ There are 3 methods to represent signed data

(i) signed magnitude representation

(ii) signed 1's complement representation

(iii) signed 2's complement representation.

→ In all above representations the +ve data is represented in normal form i.e; no complement is required but -ve data is represented in the complement given in the notation only.

→ Result for the given data is same if 1's complement operation or 2's complement operation is performed twice

Consider below table

| Representation | +5 | -5 |
|---|---|---|
| normal method | 0101 (0x) | 1101 (1x) |
| 1's Complement | 0101 (0x) | 1010 (1$\bar{x}$) |
| 2's complement | 0101 (0x) | (1011) (1$\bar{x}$+1) |

→ 1's of x is $\begin{cases} 0x & \text{if } x \geq 0 \\ 1\bar{x} & \text{if } x < 0 \end{cases}$

→ 2's of x is $\begin{cases} 0x & \text{if } x \geq 0 \\ 1\bar{x}+1 & \text{if } x < 0 \end{cases}$

Q: 1011 is signed 2's complement notation. its decimal value is

Given 2's comp

Since any of 1's (or) 2's comp is applied on same number we get original value.

So apply 2's comp

let x = 1011

$\bar{x}$ = 1100

$\bar{x}$ + 1 = 1101 → original value

i.e: -5

→ for unsigned number with size n ranges from 0 to $2^n - 1$.

Shortcut to convert decimal to binary:

→ $(26)_{10}$

~~perfect~~ ~~power~~

~~her~~ ~~het~~ hieghest power of 2 below 26 is 16

| | 16 | 8 | 4 | 2 | 1 |
|---|---|---|---|---|---|
| 26 = | 1 | 1 | 0 | 1 | 0 |

i) Signed magnitude notation:

+ve { 0000 to 0111 } +7

-ve { 1000 to 1111 } -0 to -7

Range

$(-(2^{n-1}-1))$ to $(2^{n-1}-1)$

→ zero has two notation of which '-0' is invalid

(ii) Signed 1's Complement Notation:

+ve { 0000 } +0
    { 0111 } to +7

-ve { 1000 } -7
    { 1111 } to -0

Range
$-(2^{n-1}-1)$ to $(2^{n-1}-1)$

→ Zero has 2 notations of which '-0' is invalid

(iii) Signed 2's Complement Notation:

→ It is free from negative zero.

→ Provides more range for negative data than other notations.

→ It permits 8421 weighted code while finding its decimal value.
In this process only MSB weight sign is -ve.

→ It permits sign bit expression for converting the smaller size signed data to larger size.

→ The additional padded bits are known as dummy bits/Gaurd bits.

+ve { 0000 to 0111 } 0 to +7

-ve { 1000 to 1111 } -8 to -1

-8 to +7

Range
$-2^{n-1}$ to $2^{n-1}-1$

2012
Q: The smallest -ve data that can be represented in signed 2's complement notation for 8 bit size is _____

a) 127 b) 128 c) 255 d) 256

The following data are in signed 2's complement Notation and their decimal values are:

→ $101 = (-)\overline{01} + 1 = -3$

shortcut

$$\begin{matrix} -4 & 2 & 1 \\ 1 & 0 & 1 \end{matrix} = -4+1 = -3$$

→ $1000 = (-)\overline{000} + 1 = -8$

shortcut

$$\begin{matrix} -8 & 4 & 2 & 1 \\ 1 & 0 & 0 & 0 \end{matrix} = -8$$

→ $\begin{matrix} -16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{matrix}$

$-16+1 = -15$

→ $\begin{matrix} -32 & 16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{matrix}$

$\Rightarrow -32+8+4+2 = -18$

→ $\begin{matrix} -256 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix} = -256+1 = -255$

→ $\begin{matrix} -512 & \cdots & & & & & & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix} = -1$

(If all digit are 1's in 2's Complement representation the value is 1)

i.e, It permits 8421 weighted code with MSB weight is -ve.

Now consider +2

010

here sign bit is 0

Now 010 ~ 0010 ~ 00010 . . . .

Now consider -2

$10 = (-2)_{10}$

~~$(10)_2$~~ $110 = -2$

$1110 \sim 11110 \sim \ldots$

All the padded extra bits are known as dummy bits.

i.e; It permits sign bit extension.

Q: Find no of dummy bits in

$(111010)_2$

111010

↓

Dummy bits

Q: The 16 bit data $(FFFF)_{16}$ is in signed 2's complement notation, its decimal value is ____

a) -1 b) -2 c) 10 d) none

1111 1111 1111 1111

Since all are ones value is -1

we have 15 dummy bits => original value is $-2^0$ => -1

MSD

## Problems

4) $(-539)_{10}$

$539 = 1000011011$

$-539$ -

1's comp = 0100100100

2's comp = 0100100101

1C

Problems:

2) -127

$127 = 1111111$

$\overline{127} = 0000000$

$\overline{127} + 1 = 0000001$

sign bit

1's comp = 10000000

$-127 \Rightarrow 10000001$ (2's comp) $n = 1$

$m = 2$

~~$n = 6$~~

~~$m : n = 2 : 6 = 1 : 3$~~

$m : n = 2 : 1$

4) $(-539)_{10}$

$539 = 1000011011$

$\overline{539} = 0111100100$

$\overline{539} + 1 = 0111100101$

$-539 \Rightarrow 1101111100101$

D E 5

sign bit extension

(2017 7)

x = no of distinct 16 bit integers in 2's compt

range: $-2^{15}$ to $2^{15} - 1$ ~~$\Rightarrow 2^{15} + 2^{15} - 1 = 2^{15}$~~

i.e; $12^{16}$

Y = no of distinct 16 bit integers in sign magnitude

$Y = 2^{16} - 1$

$\Rightarrow X - Y = 1$

(This can be answered directly)

## Overflow (OVFL) in signed 2's complement Notation:

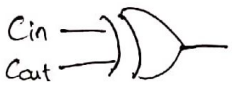
→ OVFL is known as signed CARRY

→ while adding 2 n-bit data the OVFL occurs when 'n' bits are not sufficient to store the result.

→ OVFL does not occur after adding one +ve data and one -ve data.

→ Occurance of ↑OVFL changes (disturbs) the target sign bit in the result.

→ OVFL detection




Cin = carry generated by adding data that enters in sign bit position

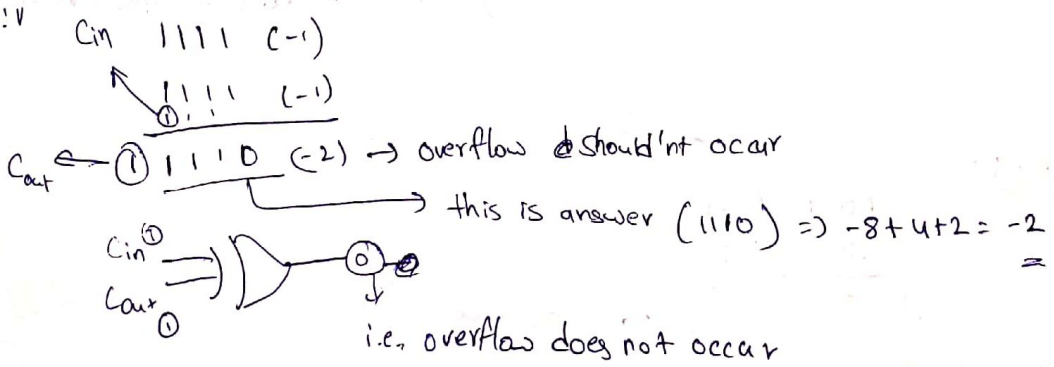
Cout = carry generated by adding both sign bits.

→ Presence of both Cin and Cout are required to detect OVFL.

Note:

* Cout participates in final result when OVFL occured only.

Eg:



Eg 2:

0 1 1 1  7
1 1 1 0  -2
① 1
① 0 1 0 1

→ 1 +ve & 1 -ve

So overflow does not occur

↳ if no overflow this 4 bits gives answers

0 → overflow does not occur

Eg 3:

0 1 1 1  +7
0 1 1 0  +6
① 1
② 1 1 0 1

1 → overflow occurs

Eg 5:

0 1 1 0  +6
1 0 1 1  -5
① 1
① 0 0 0 1

↳ result

∴ OVFL doesn't occur

Eg 4:

1 0 0 0  -8
1 0 0 1  -7
①
① 0 0 0 1

⇒ overflow occurs

Conclusion: To determine if OVFL occurs we need to know abt both $C_{in}$ & $C_{out}$

Q: After adding 2 signed 2's comp. ~~numbers~~, notation data, OVFL

a) occurs if there is $C_{in}$

b) occurs if there is $C_{out}$

c) occurs if $C_{in} = C_{out}$

✓ d) doesn't occur after adding one +ve data & one -ve data

Eg 1:

1 0 1 1  (-5)
1 0 0 0  (-8)
① 0 0 1 1  (-13)

⇒ overflow occurs

∴ $C_{out}$ participates in final result

⇒ final result 1 0 0 1 1 • i.e: $-16+2+1 = -13$

Eg:2 :

```
  1 1 1
  0 1 1 1   +7
  1 1 1 1   -1
-------------
1 0 1 1 0   +6
```

⇒ OVFL does not occur

↓

Cout do not participate in result

∴ result : 0110

Eg3 :...

```
  0 1 1 1   +7
  0 1 1 0   +6
-------------
1 1 1 0 1   +13
```

overflow occurs

final result : 01101 i.e, 13

Q: Let 'x' is the sign bit of 1st data, Y is sign bit of 2nd data and 'z' is target sign bit in the result.

The expression for the occurance of OVFL is

options

a) $xyz$

b) $xy + yz + xz$

c) $\bar{x}\bar{y}z + xy\bar{z}$ ✓

d) $x\bar{y}\bar{z} + xyz$

| x | y | Cin | ~~ovfl~~ OVFL |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 ✓ |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 ✓ |
| 1 | 1 | 1 | 0 |

S.OP : $xy\bar{C}in + \bar{x}\bar{y}Cin$

$x=0, y=0, Cin=1 \Rightarrow z=1 \Rightarrow \bar{x}\bar{y}z$

~~x=0 y=~~ $x=1, y=1, Cin=0 \Rightarrow z=0 \Rightarrow xy\bar{z}$

∴ expr is $\bar{x}\bar{y}z + xy\bar{z}$

Method 2 :

| xyz | OVFL | |
|---|---|---|
| 000 | 0 | → x=0, y=0, z=0 (two +ve gave +ve which is fine) |
| 001 | 1 | → x=0, y=0, z=1 (adding two +ve gave -ve ∴ OVFL occured) |
| 010 | 0 | two +ve & one -ve ∴ no OVFL |
| 011 | 0 | |
| 100 | 0 | |
| 101 | 0 | |
| 110 | 1 | → x=0 y=0 z=1 (adding 2 -ve gave +ve ∴ OVFL) |
| 111 | 0 | → x=1 y=1 z=1 (-ve & -ve gave -ve which is fine) |

Now S.O.P is :

$\bar{x}yz + xy\bar{z}$

Conclusion

$S_1$ x x x x . . . .  — $S_1$ – sign bit of 1st data

$S_2$ x x x x . . .  — $S_2$ – sign bit of 2nd data

S x x x x . . . .  — S – sign bit of sum of 1st & 2nd data

if ~~no~~ if $S_1$ & $S_2$ are same sign and

- if overflow not occurs S is same → $S_1$ & $S_2$
  if overflow occurs S is opposite of $S_1$ & $S_2$

# 4. Combinational Circuits

→ These are faster.

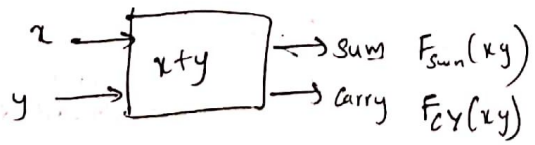
→ Present o/p depends on i/p only.

→ No memory & no clock.

→ Different combinational circuits are Adders, Subtractors, MUX, DeMux, encoder, decoder etc.

→ MUX is known as universal combinational circuit.

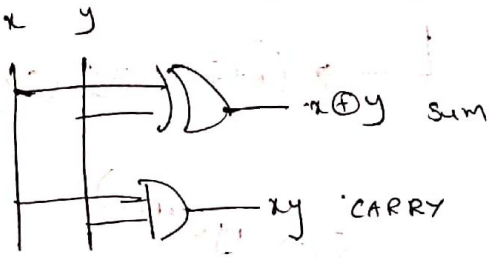
## Half Adder



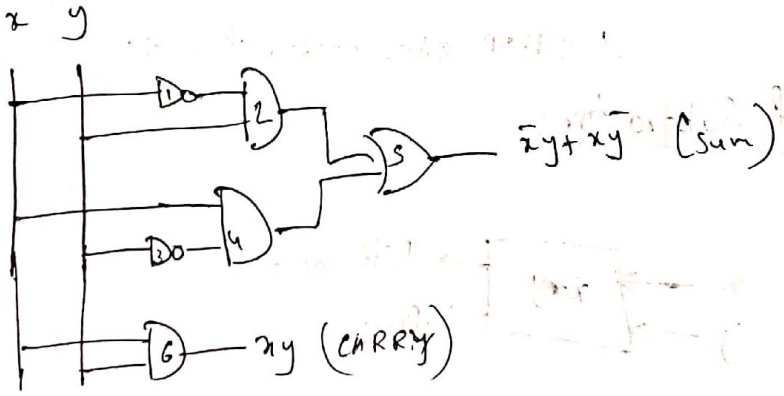
| x | y | S | CY |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$F_{sum}(xy) = \bar{x}y + x\bar{y} = x \oplus y$

$F_{cy}(xy) = xy$

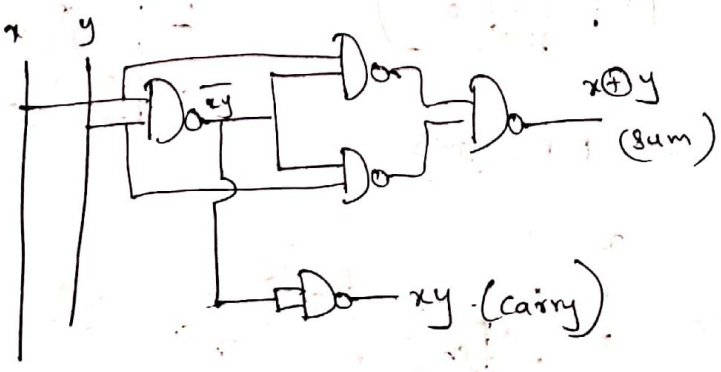



HA with basic gates




Here we need 6 basic gates

HA with NAND gates




∴ Here we need 5 NAND gates

## HA with NOR gates

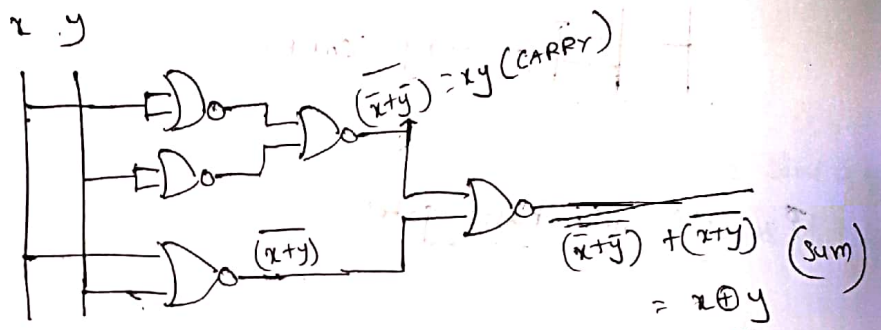
$$\text{Sum} = x \oplus y = \bar{x}y + x\bar{y}$$

$$= \overline{xy + \bar{x}\bar{y}}$$

$$= \overline{(\overline{xy}) + \bar{x}\bar{y}}$$

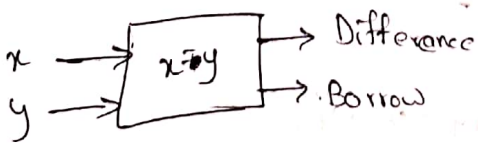
$$= (\overline{\bar{x}+\bar{y}}) + (\overline{x+y}) \longrightarrow \text{Sum}$$

$$\hookrightarrow (\overline{\bar{x}+\bar{y}}) = xy \ (\text{C.y})$$




∴ 5 NOR gates needed for HA

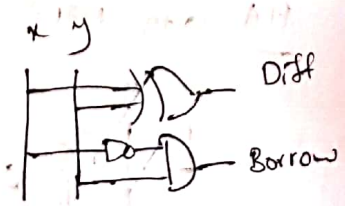
## Half Subtractor:



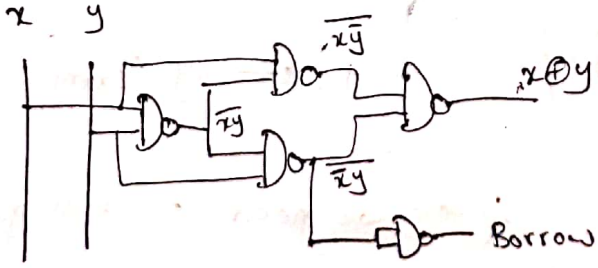

| x | y | Diff | Borrow |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

Diff = $x \oplus y$

Borrow = $\bar{x}y$




## HS with NAND gates




∴ 5 NAND gates for HS

$\overline{\bar{x}\bar{y} \cdot x}$

$xy + \bar{x}$

$\bar{x} + y$

$\overline{\bar{x}\bar{y}}$

$(\overline{\bar{x}+\bar{y}})$

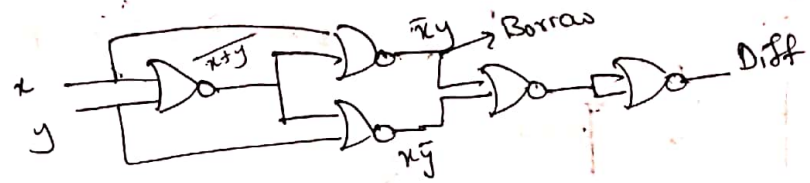
$\overline{xy} \cdot y$

$xy + \bar{y}$

$(xy)(y\bar{y})$

$xy = \bar{x}$

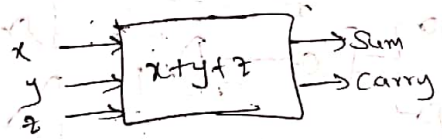
## HS with NOR gates




Note:

Min no of NAND/NOR gates required for Half adder or Half Subtractor is 5.

## Full Adder:



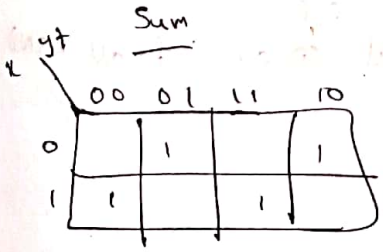

| xyz | Sum | carry |
|---|---|---|
| 000 | 0 | 0 |
| 001 | 1 | 0 |
| 010 | 1 | 0 |
| 011 | 0 | 1 |
| 100 | 1 | 0 |
| 101 | 0 | 1 |
| 110 | 0 | 1 |
| 111 | 1 | 1 |

$Sum(xyz) = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

$Sum(xyz) = \Sigma m(1,2,4,7)$

$\& \quad \Sigma(xyz) = \Sigma m(3,5,6,7)$

Sum




$\bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

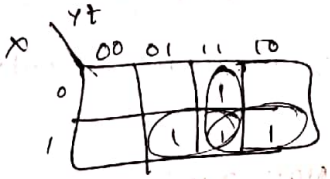
$\bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$

$\bar{x}(y \oplus z) + x(y \odot z)$

$\bar{x}(y \oplus z) + x\overline{(y \oplus z)}$

$x \oplus y \oplus z$

CARRY



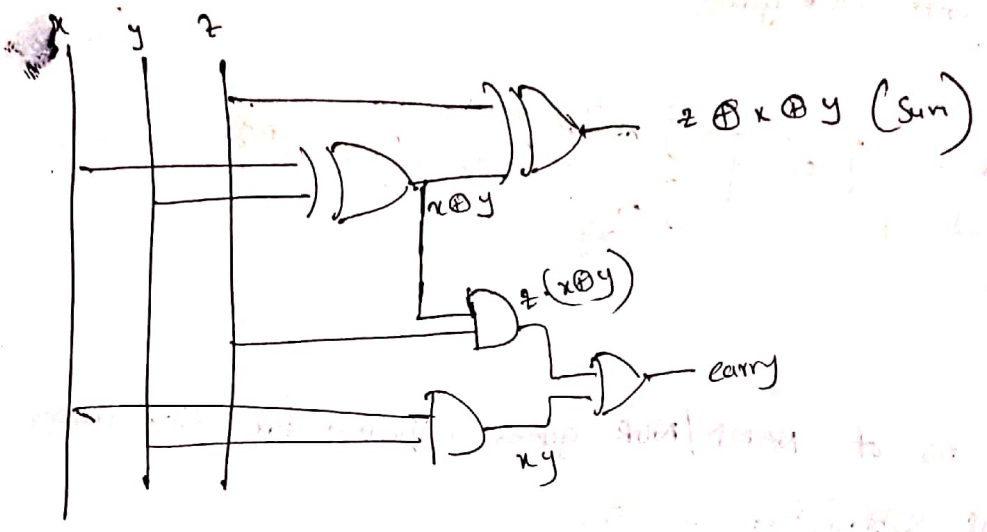

$yz + xz + xy$

(or)

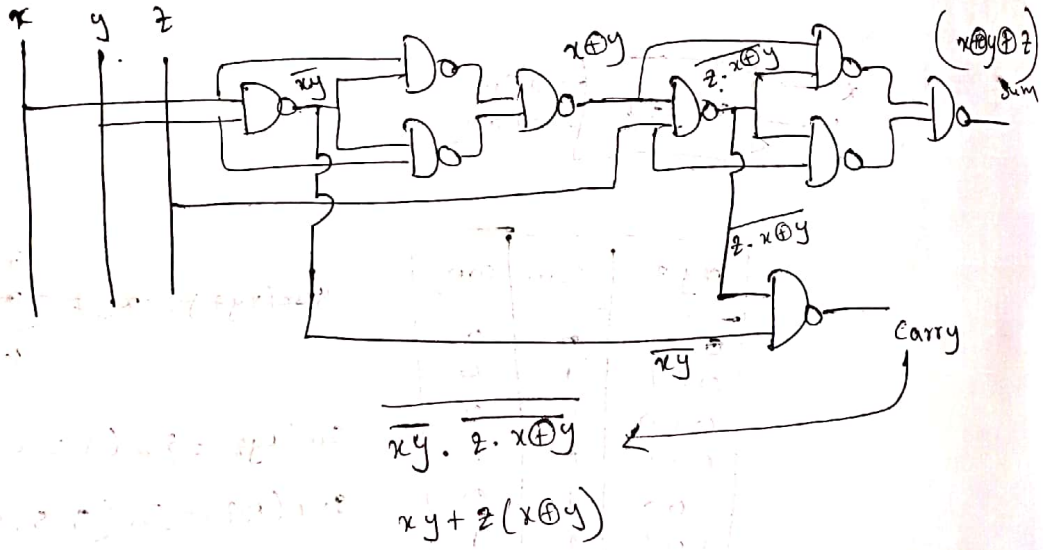
$\bar{x}yz + x\bar{y}z + xy$ (This is written directly from minterms)

$z(x \oplus y) + xy$

Here $z(x \oplus y) + xy$ is better to use.



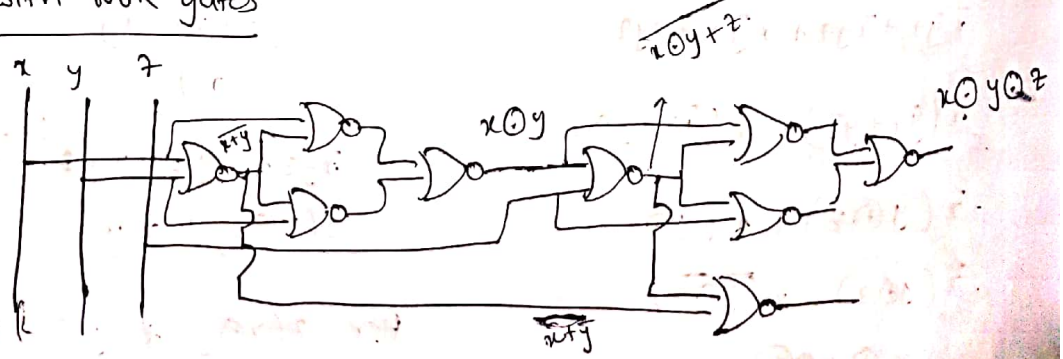

Full Adder with NAND gates




Note :

Above circuit produces same functional O/P even if all NAND gates are replaced with NOR gates.

FA with NOR gates



Sum = $x \odot y \odot z$

$= (x \odot y) \oplus \bar{z}$ (Single bubbled ExNOR is Ex OR)

$= \overline{(x \oplus y)} \oplus \bar{z}$ (1

$= x \oplus y \oplus z$. (double bubbled Ex OR is normal Ex OR)

At 5th gate

o/p : $\overline{x \odot y + z}$

$= \overline{xy + \bar{x}\bar{y} + z}$

At 9th gate

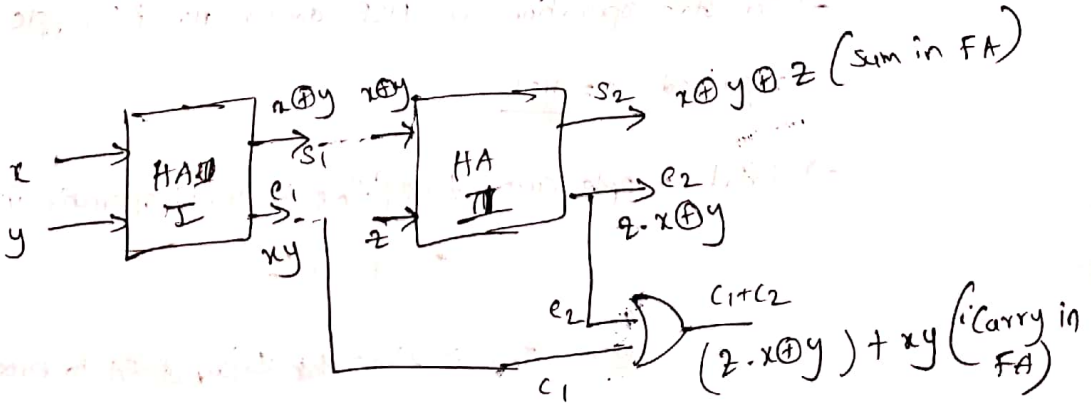
$\overline{\left(\overline{z + xy + \bar{x}\bar{y}}\right) + (\overline{x+y})}$

$\overline{\left(\overline{z + xy + \bar{x}\bar{y}}\right) \cdot (x+y)}$

$= yz + xz + xy$ (expression obtained from carry k-map)

## Full Adder with half adder:

* Here we use 2 half adder & 1 OR gate to obtain 1 full adder




## n bit adder:

→ It is used to add 2 n bit data.

→ FA is known as one full bit adder.

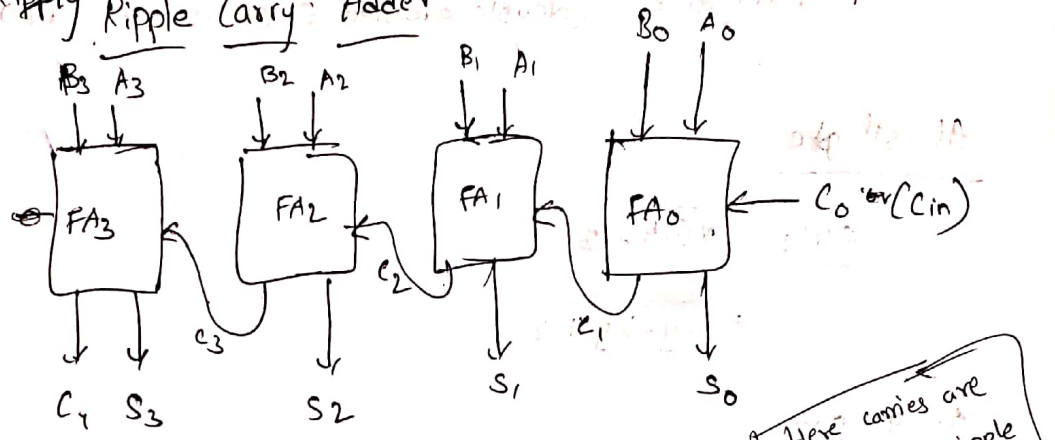
→ HA is known as one half bit adder.

(2023)

Q To add 2 n bit data (without Cin) min no of of HA required is 1 and min no. of FA req is n-1.

→ Here Cin refers to initial carry.

Ripply Ripple Carry Adder




Here carries are known as ripple carries

final result: $C_4 S_3 S_2 S_1 S_0$.

Note:

In above diagram all $A_i, B_i$ & $C_0$ are available directly (ie: no delay)

for n bit Ripple carry adder:

→ After completion of operation in MSB adder ($FA_{n-1}$) the final result will be stabilized

→ To start operation in MSB adder no of ripple carries required is = n-1

→ Total ripple carry time (time to start operation in MSB) = $(n-1) \times T_{cy}$

Here $T_{cy}$ is time delay of FA to produce carry

→ $T_{worst} = ((n-1) \times T_{cy}) + \max(T_{sum}, T_{cy})$

Here $T_{worst}$ is max time to produce stabilized output

$T_{sum}$ is time taken by FA to produce sum.

## Problems

5) $(16-1) \times 12 + 15 = 195$ ns → worst case delay

- $(16-1)$ → total carries to produce start operation in MSBFA
- $12$ → Time to start operation in MSB
- $15$ → $\text{Max}(T_{sum}, T_{CY})$

(2016)

4) $A = (1)_{10}$

$i.e_{,} \quad = (00000001)_2$ (latency means time delay)

*longest latency is required when CARRY is generated in MSB adder (i.e when longest data i/p is applied)

$A = 00000001$

$B = 11111111$

$\quad\quad (1)1111111$

$\therefore B = (-1)_{10}$

2) HA with XOR & AND

FA with 2HA & 1OR

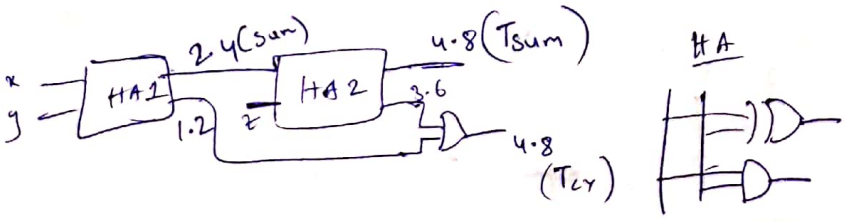
$t_{XOR} = 2t_{AND} = 2t_{OR}$

$t_{AND} = t_{OR} = 1.2\,\mu s$

$t_{XOR} = 2.4\,\mu s$

4-bit ripple carry adder with 4FA

Top Total propagation delay = ? $\mu s$




HA



~~Since we are using 4 FA~~

Here both carry & sum are generated exactly after 4.8 $\mu s$

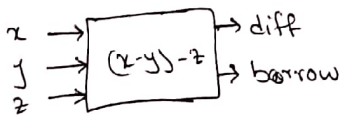
we are using 4FA => ~~4 x 4.8 = 19.2 $\mu s$~~ $(4-1) \times 4.8 + 4.8 = 19.2\,\mu s$

Method 2:

FA3 FA2 FA1 FA0

07/09/19

## Full Subtractor:



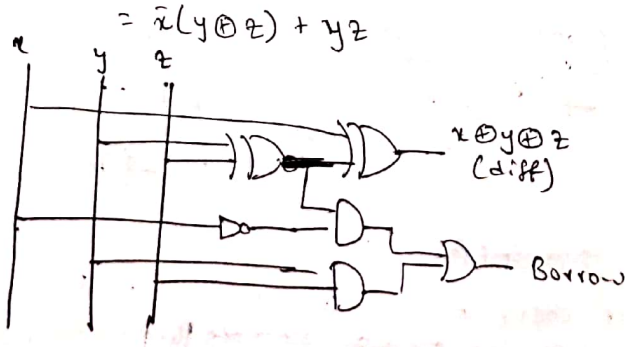

| x y z | D | B |
|---|---|---|
| 0 0 0 | 0 | 0 |
| 0 0 1 | 1 | 1 |
| 0 1 0 | 1 | 1 |
| 0 1 1 | 0 | 1 |
| 1 0 0 | 1 | 0 |
| 1 0 1 | 0 | 0 |
| 1 1 0 | 0 | 0 |
| 1 1 1 | 1 | 1 |

$F_{Diff}(xyz) = \Sigma_m(1,2,4,7)$

$= x \oplus y \oplus z$

$F_{Borrow}(xyz) = \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xyz$

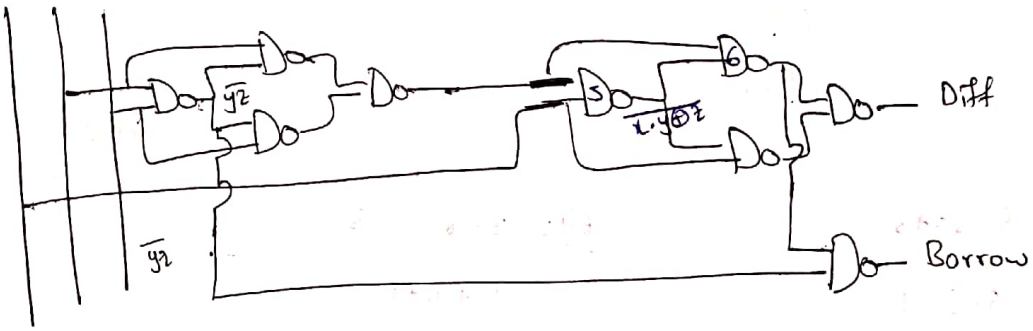
$= \bar{x}(y \oplus z) + yz$



## Full Subtracter with NAND gates:

Borrow = $\bar{x}(y \oplus z) + yz$

consider it as $\overline{\overline{\bar{x}(y \oplus z) + yz}} = \overline{\overline{\bar{x}(y \oplus z)} \cdot \overline{yz}}$




At 6th gate :

$\overline{(y \oplus z) \cdot x \cdot \overline{y \oplus z}}$

$= \overline{(y \oplus z) \cdot (\bar{x} + \overline{y \oplus z})}$

$= (y \oplus z) \cdot \bar{x} + y \oplus z \cdot \overline{y \oplus z}$

$= \overline{\bar{x} \cdot y \oplus z}$

Note :

The above diagram produces same functioning even if all NAND gates are replaced with NOR gates

## Binary Substraction:

→ It is performed with 2's complement Addition

i.e, $A - B = A + \bar{B} + 1$

→ During this operation if there is a carry then result sign is +ve and +ve result is available in normal form.

→ If there is no carry then result sign is -ve and -ve result is available in 2's complement form.

Q: Min no of 2 i/p NAND / NOR gates req to implement Full Adder/ Full Subtractor is 9

Here $A-B$

$= A + (-B)$

$= A + \bar{B} + 1$

Eg: $6-3 = +3$

$6 + \bar{3} + 1$

$6 : 110$

$\bar{3} : 100$

$1$

$1011$

$3$

(No Carry generated ∴ 011 is answer)

Eg: $3-6 = -3$

$3 + \bar{6} + 1$

$3 : 011$

$\bar{6} : 001$

$1$

$1001$ → This 2's comp form (∵ no carry)

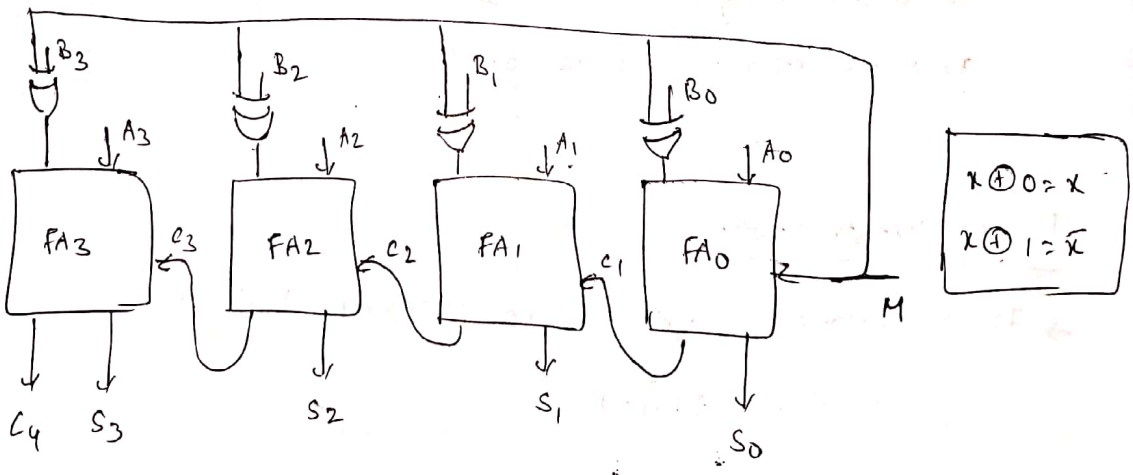
∴ $101 = -4+0+1 = -3$

(or) $101 = 110$ (2 1's comp)

$= 111$ (original)

$\therefore$ i.e. $-3$

4 bit Adder/ Subtractor :




→ Here M is called Mode control

→ If M=0 the above circuit acts as Full Adder

$A_3\ A_2\ A_1\ A_0$

$B_3\ B_2\ B_1\ B_0$ } adder

→ If M=1 the circuit acts as full subtractor

A: $A_3 A_2 A_1 A_0$

(2's comp) { (1's) $\bar{B}$: $\bar{B}_3 \bar{B}_2 \bar{B}_1 \bar{B}_0$ ; 1 } Subtractor

## Multiplexer

→ It is also known as universal combinational circuit, Data selector, Many to one selector and parallel to Serial Converter.

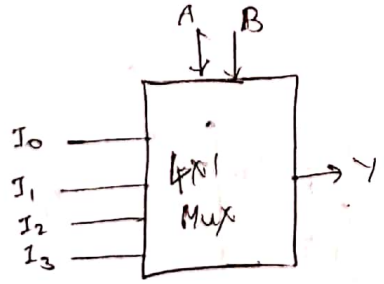
Standard size is $2^n \times 1$, $2^n$ = no of data inputs with n selection lines

→ Selection lines are used to select only one input to output line.

→ Different sizes are $2\times1, 4\times1, 8\times1, \ldots 2^n\times1$

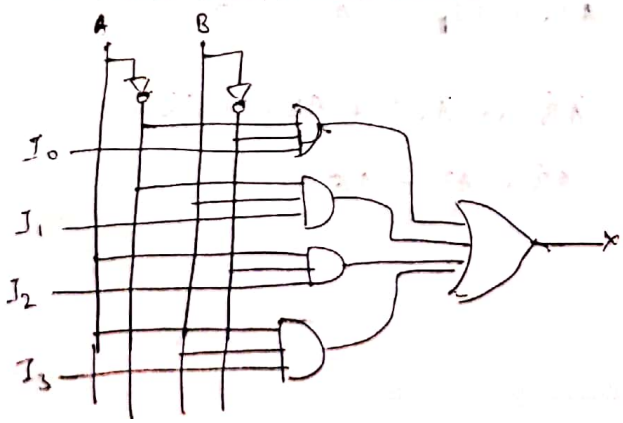
Q: No of selection lines required for $1024\times1$ MUX is 10

Q: No of selection lines require for $N\times1$ MUX is $\log_2 N$




| $S_1$ | $S_0$ | Y |
|---|---|---|
| 0 | 0 | $I_0$ |
| 0 | 1 | $I_1$ |
| 1 | 0 | $I_2$ |
| 1 | 1 | $I_3$ |

$Y = \bar{A}\bar{B}I_0 + \bar{A}BI_1 + A\bar{B}I_2 + ABI_3$



For designing 4x1 Mux

No of AND gates = 4

No of OR gates = 1

No of NOT = 2

~~No of 0's~~

Size of OR gate = 4

Size of each AND gate = 2 + 1
(S, I)

For designing $2^n$x1 Mux

No of AND gates $= 2^n$

No of OR gates = 1

No of NOT gates = n

Size of OR gates $= 2^n$

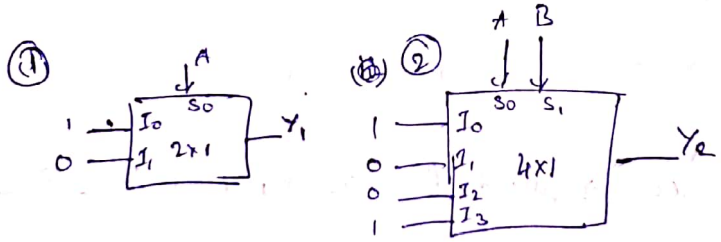
~~No of~~

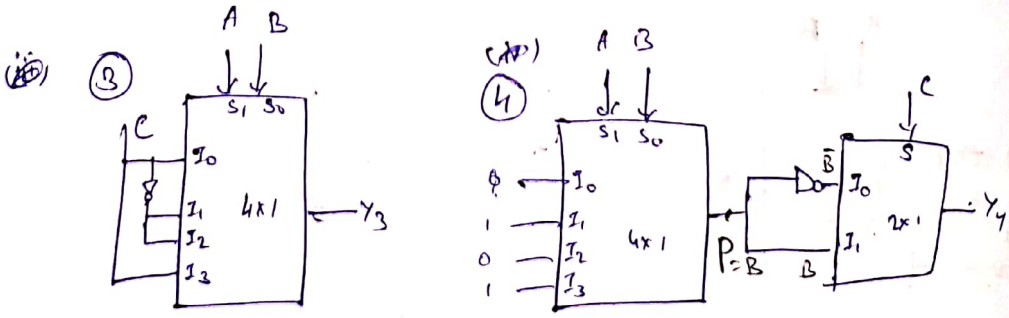
size of each AND gate = n+1

Previous Q: No of inputs req for each AND gate to design 32x1 Mux is 6

5+1 = 6

Eg: What is the o/p at Y in the given below




① $Y_1 = \bar{A} I_0 + A I_1 = \bar{A}$

| A | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

② $Y_2 = \bar{A}\bar{B} I_0 + \bar{A} B I_1 + A \bar{B} I_2 + A B I_3$

$= \bar{A}\bar{B} + AB = A \odot B$

③

| AB | Y |
|---|---|
| 00 | C |
| 01 | $\bar{C}$ |
| 10 | $\bar{C}$ |
| 11 | C |

$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

$\Sigma m(1,2,4,7) = A \oplus B \oplus C$

④

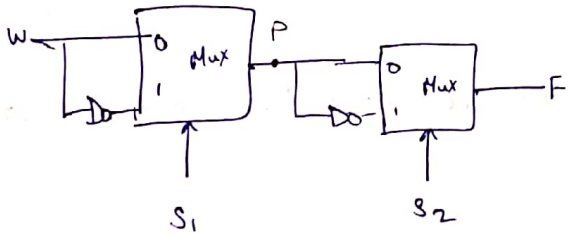
| AB | P |
|---|---|
| 00 | 0 |
| 01 | 1 |
| 10 | 0 |
| 11 | 1 |

$P = \bar{A}B + AB = B$

$\Rightarrow Y_4 = \bar{C}B + CB = B \odot C$

## Problems:

① (2014)




$P = \bar{S}_1 W + S_1 \bar{W} = S_1 \oplus W$

$F = \bar{S}_2 P + S_2 \bar{P} = S_2 \oplus P = S_2 \oplus S_1 \oplus W$

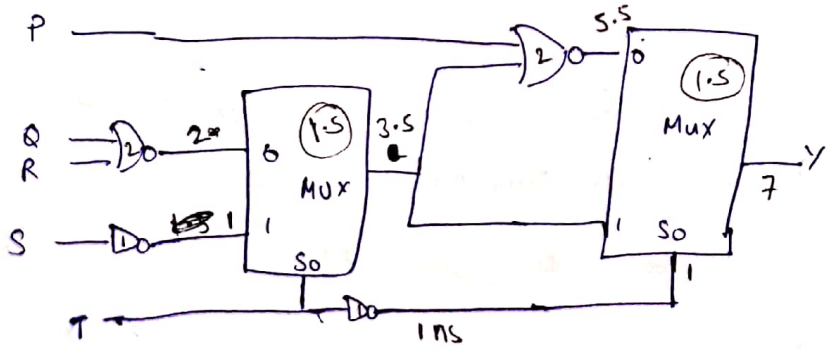
$F = W \oplus S_1 \oplus S_2$

EEE 2016

③

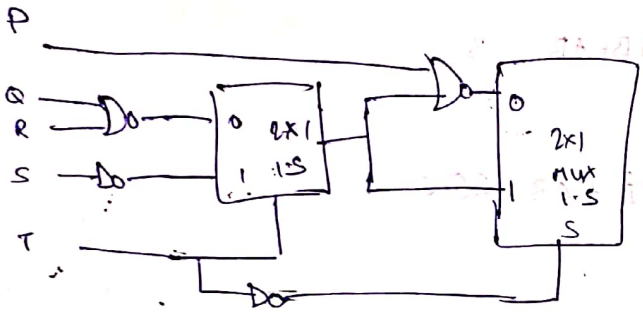
$t_{NOR} = 2 \quad t_{mux} = 1.5 \quad t_{not} = 1ns$

P, Q, R, S, T at same instant applied



| To | Data Travel path. |
|---|---|
| 0 | $T_{NOR} + T_{MUX} + T_{MUX} = 5ns$ |
| 1 | $T_{NOT} + T_{MUX} + T_{NOR} + T_{MUX} = 6ns$ |

Consider the circuit as below




## Mux as function selector

→ Generally there are 2 methods used to implement n variable function

(i) with $2^n \times 1$ size

(ii) $2^{n-1} \times 1$ size

→ while implementing with $2^n \times 1$ size MUX all functional inputs are connected to selection lines and data inputs of Mux can be 0's & 1's

→ while implementing the fn with $2^{n-1} \times 1$ size mux (n-1) functional inputs are connected to selection lines and data inputs for Mux can be 0's, 1's, free input, complement of free input.

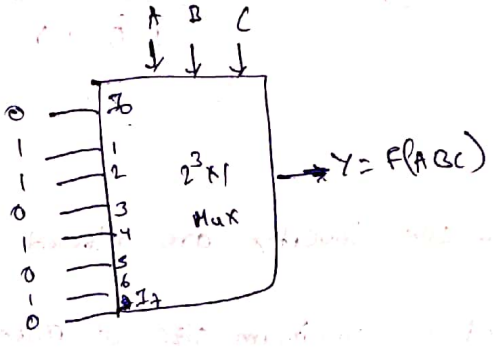
Note:-

while implementing n variable fn with $2^{n-1} \times 1$ size mux, NOT gate is not needed for all functions (like carry in F/A) but NOT is gate is needed for only some functions like (Sum in FA)

Eg: Implement $F(ABC) = \Sigma_m (1,2,4,6)$ with Mux

Using $2^3 \times 1$ Mux $(2^n \times 1)$:

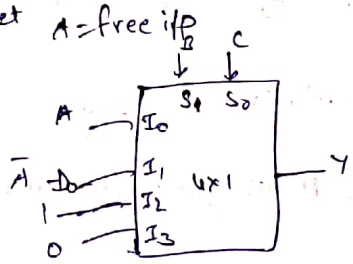
| A B C | F |
|---|---|
| 0 0 0 | 0 |
| 0 0 1 | 1 |
| 0 1 0 | 1 |
| 0 1 1 | 0 |
| 1 0 0 | 1 |
| 1 0 1 | 0 |
| 1 1 0 | 1 |
| 1 1 1 | 0 |




Using $2^{2-1} \times 1$ Mux $(2^{n-1} \times 1)$:

| ABC | F |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 1 |
| 111 | 0 |

Let A = free i/p

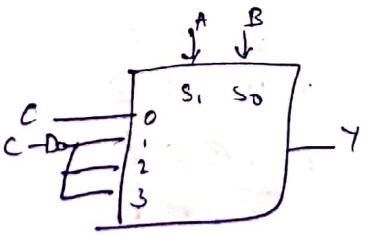



| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $\bar{A}$ | 0 | (1) | (2) | 3 |
| A | (4) | 5 | (6) | 7 |
| | A | $\bar{A}$ | 1 | 0 |

Verification

| BC | Y |
|---|---|
| 00 | A → $A\bar{B}\bar{C}$ = 100 (4) |
| 01 | $\bar{A}$ → $\bar{A}\bar{B}C$ = 0001 (1) |
| 10 | 1 → $B\bar{C}$ ($AB\bar{C} + \bar{A}B\bar{C}$) (6,2) |
| 11 | 0 |

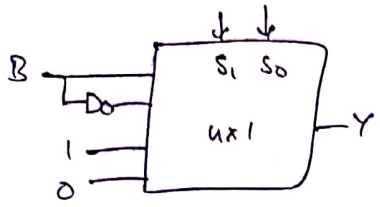
let c = free inpu




| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $\bar{C}$ | 0 | (2) | (4) | (6) |
| C | (1) | 3 | 5 | 7 |
| | C | $\bar{C}$ | $\bar{C}$ | $\bar{C}$ |

| A B | Y |
|---|---|
| 00 | C : $\bar{A}\bar{B}C$ (1) |
| 01 | $\bar{C}$ |
| 10 | $\bar{C}$ } (2,4,6) |
| 11 | $\bar{C}$ |

Let B = free i/p




| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $\bar{B}$ : | 0 | 1 | 4 | 5 |
| B : | 2 | 3 | 6 | 7 |
| | B | $\bar{B}$ | 1 | 0 |

Q: One mux & one invertex are allowed to implement n variable function. what is minimum size of mux needed

(i) $2^n \times 1$ (ii) $2^{n-1} \times 1$ (iii) $2^{n+1} \times 1$ (iv) None

→ for $2^n \times 1$ Mux to implement n variable function:

→ Implementation is easy.

→ Requires more gates compared to $2^{n-1} \times 1$ Mux implementation

→ For $2^{n-1} \times 1$ Mux to implement n variable function

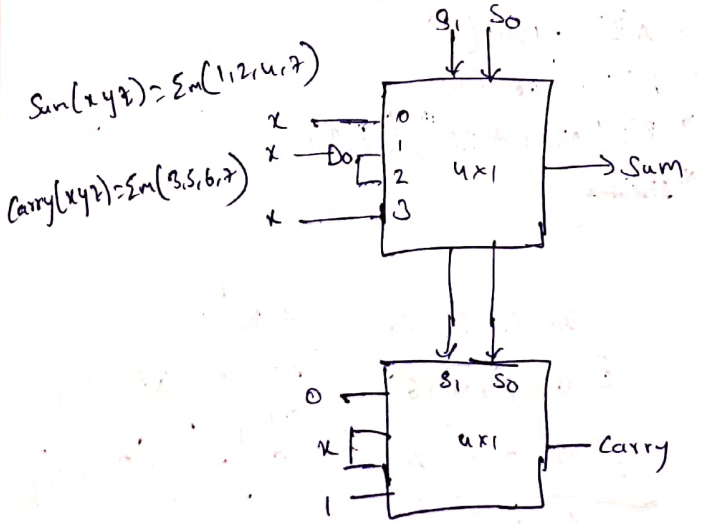
→ Implementation is complex.

→ Requires less gates.

Eg: Implement FA with 4x1 matrices

$Sum(x,y,z) = \Sigma m(1,2,4,7)$

$Carry(x,y,z) = \Sigma m(3,5,6,7)$




| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $\bar{x}$ | 0 | 1 | 2 | 3 |
| x | 4 | 5 | 6 | 7 |
| | x | $\bar{x}$ | $\bar{x}$ | x |

| | $I_0$ | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|---|
| $\bar{x}$ | 0 | 1 | 2 | 3 |
| x | 4 | 5 | 6 | 7 |
| | 0 | x | x | 1 |

Here x is $S_2$ (i.e., 3rd bit in addition)

## Logic gates with 2x1 Muxes:

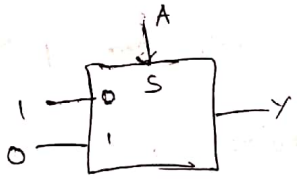
→ It is like function implementation with $2^{n-1}$ muxes.

(2003)

Q: Min no of 2x1 muxes needed to implement 2 i/p XOR gate is 2 and 2 i/p AND gate is 1

### NOT gate:

| A | Y |
|---|---|
| 0 | 1 |
| 1 | 0 |

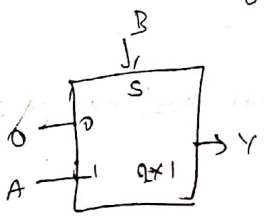



### AND gate

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| | $I_0$ | $I_1$ |
|---|---|---|
| $\bar{A}$ | 0 | 1 |
| A | 2 | 3 |

0 A

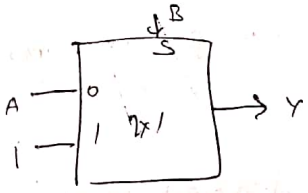



### OR gate

$f(AB) = \Sigma m(1,2,3)$

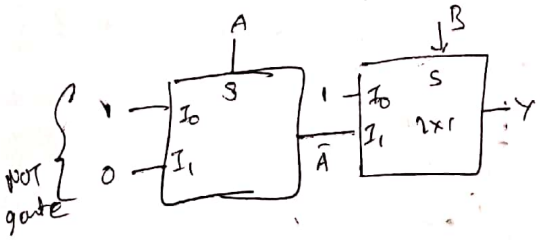
| | | |
|---|---|---|
| $\bar{A}$ | 0 | 1 |
| A | 2 | 3 |

A 1




### NAND gate

$f(AB) = \Sigma m(0,1,2)$




| | | |
|---|---|---|
| $\bar{A}$ | 0 | 1 |
| A | 2 | 3 |

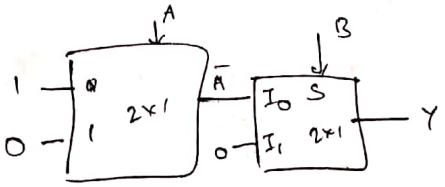
1 $\bar{A}$

↓

require not gate

### NOR gate

$f(AB) = \Sigma m(0)$




| | | |
|---|---|---|
| $\bar{A}$ | 0 | 1 |
| A | 2 | 3 |

$\bar{A}$ 0

## XOR

$f(AB) = \Sigma_m(1,2)$

| | $I_0$ | $I_1$ |
|---|---|---|
| $\bar{A}$ | 0 | (1) |
| $A$ | (2) | 3 |
| | $A$ | $\bar{A}$ |

∴ require two Muxes

## XNOR

$f(A) = \Sigma_m(0,3)$

| | $I_0$ | $I_1$ |
|---|---|---|
| $\bar{A}$ | (0) | 1 |
| $A$ | 2 | (3) |
| | $\bar{A}$ | $A$ |

∴ requires two Muxes

→ All basic 2-i/p gates require only 1 Mux.

→ All universal & derived gates require 2 Muxes.

## Enable input in a Mux:

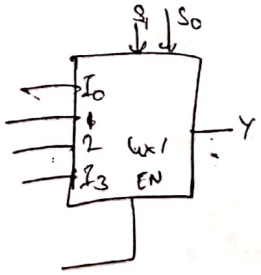
→ Some Muxes are designed with this special control input known as enable input.

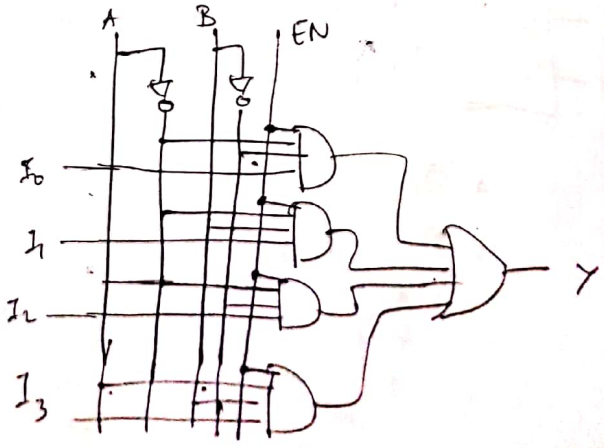
→ Before going to perform any operation the mux must be in enabled condition.

→ If mux is disabled then its o/p will be forced to '0' regardless of data inputs & selection lines.

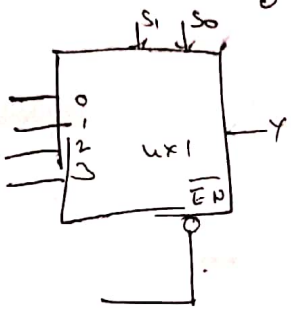
### Active high enable:

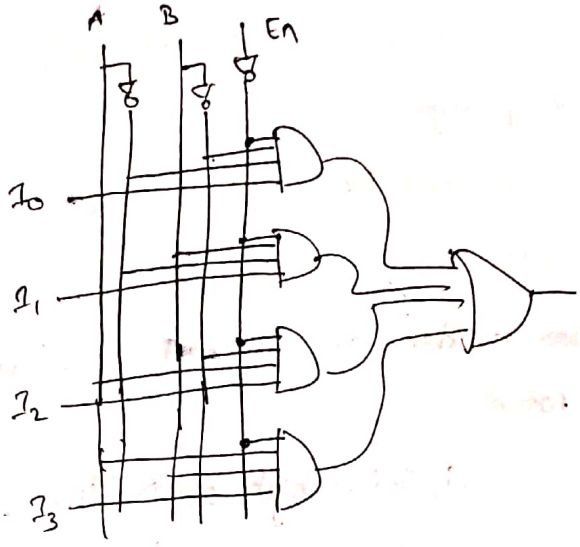
'1' for enable ; '0' for disable



Active low Enable. '0' for enable '1' for disable




## Demux :

→ It is also known as Data distributor, one to many selector, serial to parallel selector.

Standard size is $1 \times 2^n$

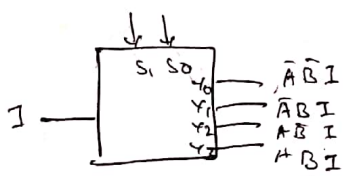
$2^n$ = no of o/p with 'n' selection lines

→ Selection lines are used to select the input 'I' to one of the inputs.

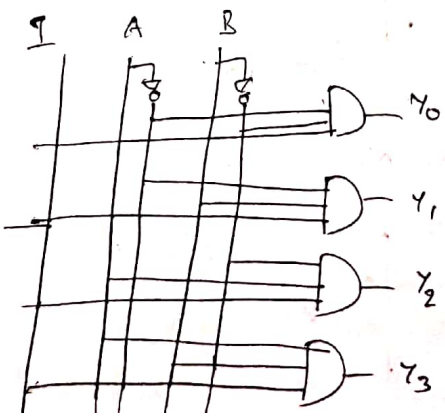
→ Different sizes are $1\times2, 1\times4, 1\times8, \ldots 1\times2^n$

Q: No of selection lines req for designing 1x256 demux is 8.

### 1x4 Demux




| AB | $Y_0$ | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|---|
| 00 | I | 0 | 0 | 0 |
| 01 | 0 | I | 0 | 0 |
| 10 | 0 | 0 | I | 0 |
| 11 | 0 | 0 | 0 | I |



# Decoder:

→ It is used for memory addressing and chip select application in memory organization.

→ No selection lines.

→ Standard size is $n \times 2^n$ (@max)

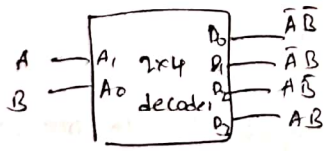
→ ~~Dec~~ Decoders are divided in 2 types that depends on its output nature.

(i) Active high o/p Decoder → only one o/p bit is high

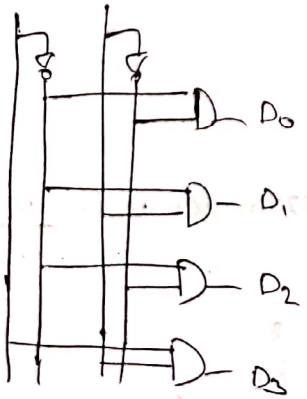
(ii) Active low o/p Decoder → only one o/p bit is low

→ whatever the size of the decoder and nature of it's o/p, always only one o/p bit is different to all other o/p bits.

## Active high o/p decoder.

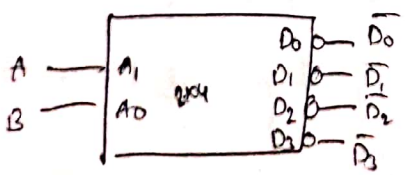



| A B | $D_0$ | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|---|
| 0 0 | 1 | 0 | 0 | 0 |
| 0 1 | 0 | 1 | 0 | 0 |
| 1 0 | 0 | 0 | 1 | 0 |
| 1 1 | 0 | 0 | 0 | 1 |




| A B C D E | High o/p give by. |
|---|---|
| 1 1 0 1 1 | $D_{27}$ |

## Active low o/p Decoder




| A B | $\bar{D_0}$ | $\bar{D_1}$ | $\bar{D_2}$ | $\bar{D_3}$ |
|---|---|---|---|---|
| 0 0 | 0 | 1 | 1 | 1 |
| 0 1 | 1 | 0 | 1 | 1 |
| 1 0 | 1 | 1 | 0 | 1 |
| 1 1 | 1 | 1 | 1 | 0 |

## Decoder as function selector:

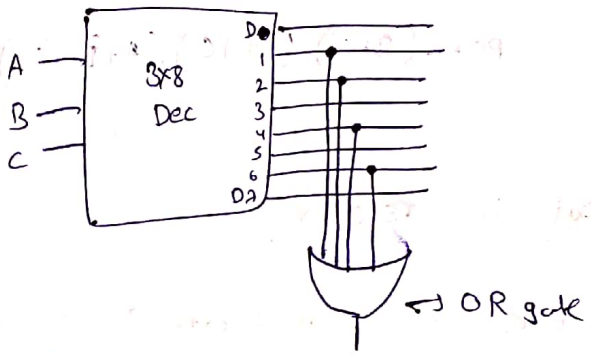
→ Generally $n \times 2^n$ size decoder is used to implement one 'n' variable function.

→ All functional inputs are connected to decoder input and minterms are conneted as input for OR gate / NAND gate

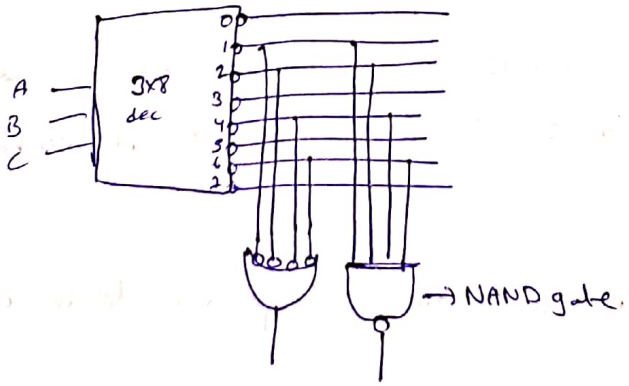
Eg: Implement $f(ABC) = \Sigma m(1,2,4,6)$ with Decoder

Active high O/P decoder

| ABC | F |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 1 |
| 111 | 0 |

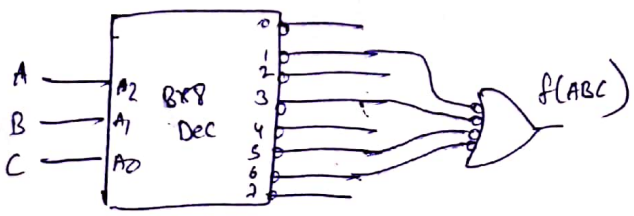



Active low O/P decoder




(2008)

Q The minimized POS expression for the given below diagram




a) $(A+C)(B+C)(\bar{A}+\bar{B}+\bar{C})$

b) $(\bar{A}+\bar{C})(\bar{B}+\bar{C})(A+B+C)$

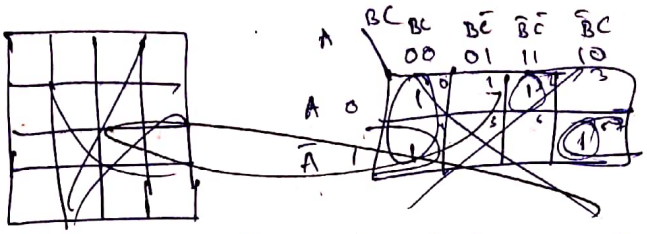
c) $\bar{B}C + \bar{A}C + AB\bar{C}$

d) None

$f(ABC) = \Sigma m(1,3,5,6) = \Pi M(0,2,4,7)$

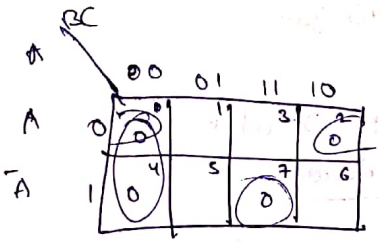
POS expression : ~~$ABC + A\bar{B}C +$~~

$(A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

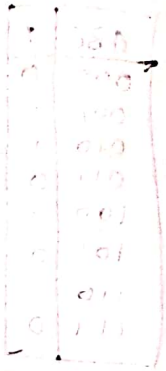



POS: $(B+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}C)$

(1,4) (4) (4,8)




pos: $(B+C)(\bar{A}+C)(\bar{A}+\bar{B}+\bar{C})$



## Enable input in decoder:

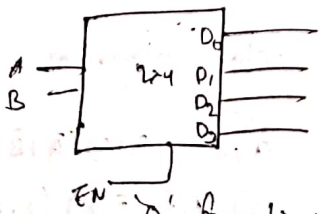
→ Some decoders are designed with special input known as enable input.

→ Before going to perform any operation the decoder should be enabled.

→ If active high o/p decoder is disabled all o/p bits will be forced to low and if active low o/p decoder is disabled all o/p bits will be forced to high.

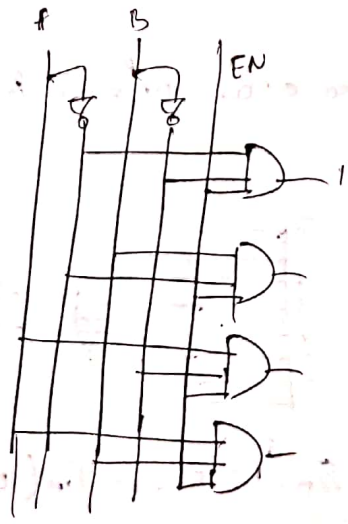
→ This enable input is used to implement larger size decoder with smaller size decoders.

Active high o/p decoder

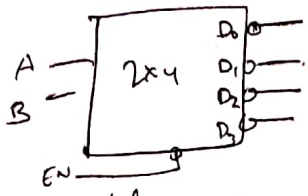



'0' for disable

'1' for enable

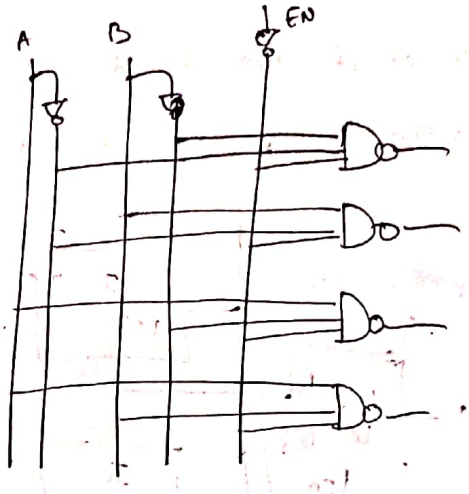


## Active low o/p decoder



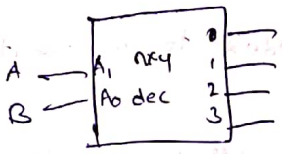

'0' for enable

'1' for disable



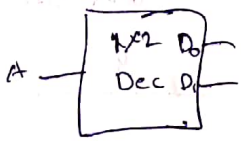

Ex: Implement 2x4 decoder with 1x2 decoders

### Target size



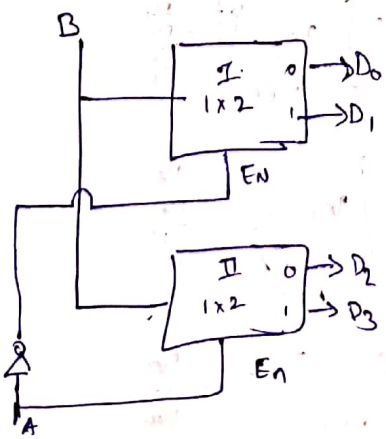

| AB | $D_0$ $D_1$ $D_2$ $D_3$ |
|---|---|
| 00 | 1 0 0 0 |
| 01 | 0 1 0 0 |
| 10 | 0 0 1 0 |
| 11 | 0 0 0 1 |

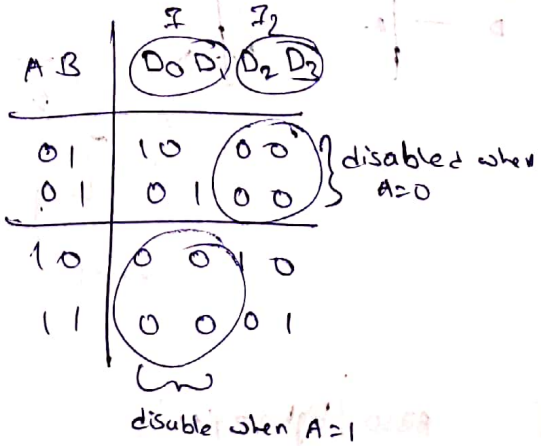
### Basic size



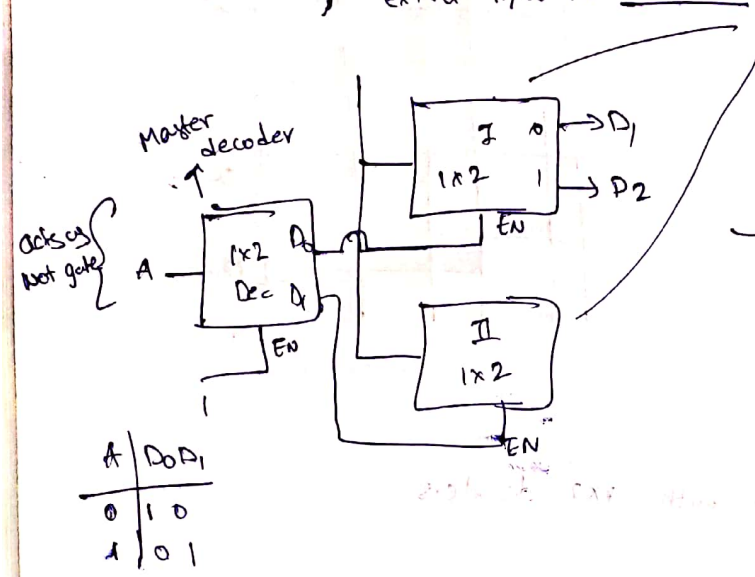

| A | $D_0$ $D_1$ |
|---|---|
| 0 | 1 0 |
| 1 | 0 1 |

## 2x4 decoders with 1x2



Q: Min no of 1x2 decoders needed to implement 2x4 decoder without using extra H/w is 3




| A | D0 D1 |
|---|---|
| 0 | 1 0 |
| 1 | 0 1 |

| AB | D0 D1 D2 D3 |
|---|---|
| 00 | 1 0 0 0 |
| 01 | 0 1 0 0 |
| 10 | 0 0 1 0 |
| 11 | 0 0 0 1 |

disabled when A=0

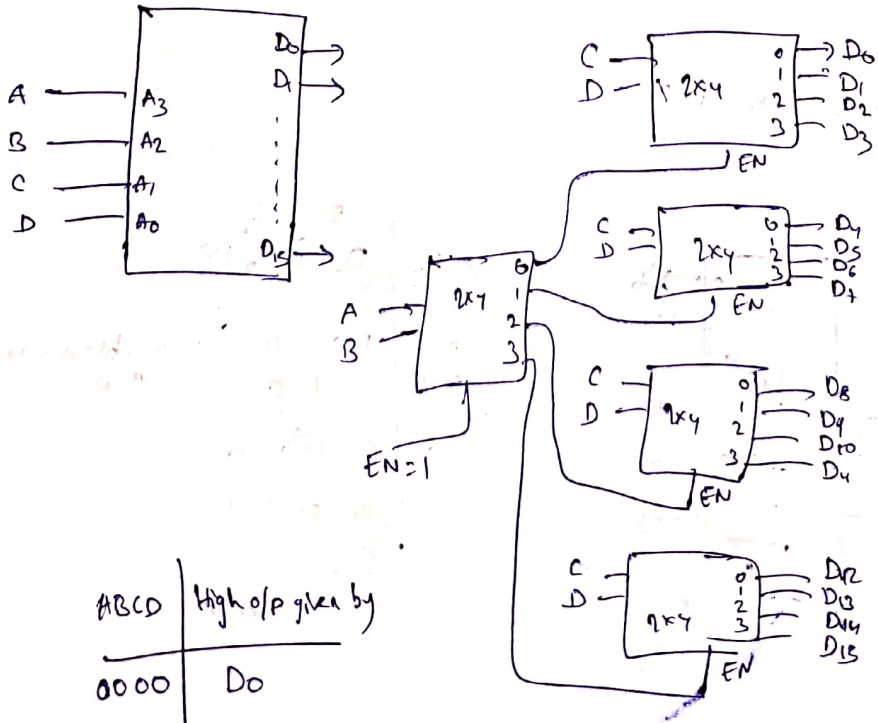
disable when A=1

2007

Q Min no of ~~size decoders~~ 8x64 Size decoders needed to implement 3x8 Size decoder is 9

$\frac{6x64^8}{3x8} = 8$, 1 Master decoder

Ex: Implement 4x16 decoder with 2x4 decoder




| ABCD | High o/p given by |
|---|---|
| 0000 | D0 |
| 0111 | D7 |
| 1001 | D9 |
| 1111 | D15 |

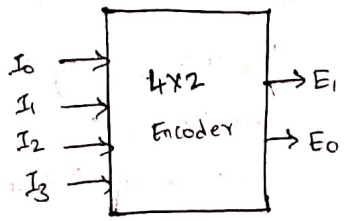
## Encoder:

→ It performs decoder's inverse function.

Standard size is $2^n \times n$

→ Always only one high signal is permitted at output.

→ Different sizes are $2\times1, 4\times2, 8\times3 \ldots 2^n \times n$

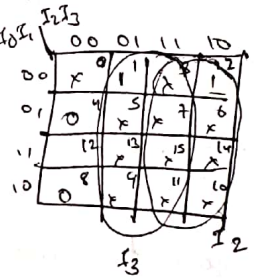



| $I_0$ $I_1$ $I_2$ $I_3$ | $E_1$ $E_0$ |
|---|---|
| 1 0 0 0 | 0 0 |
| 0 1 0 0 | 0 1 |
| 0 0 1 0 | 1 0 |
| 0 0 0 1 | 1 1 |

$E_1 = I_2 + I_3$

$E_0 = I_0 + I_4$

for $E_1$:



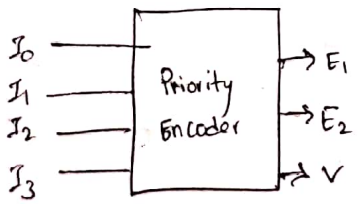

→ If i/p contains more than one high signal (Ex: 1100) then it is invalid i/p.

## Priority Encoder:

→ It permits one or more high signals at i/p.

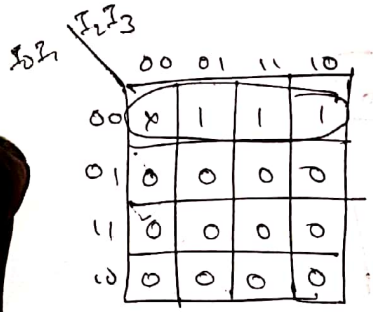
→ It is designed with one special o/p bit(V).

→ Output of the encoder becomes valid if V=1 only.



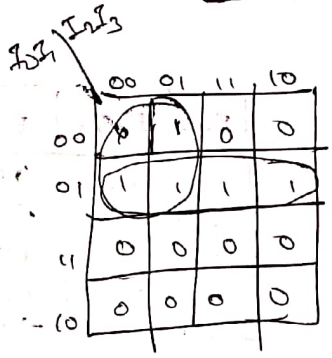
| | $I_0$ $I_1$ $I_2$ $I_3$ | $E_1$ $E_0$ | V |
|---|---|---|---|
| 0 | 0 0 0 0 | x x | 0 (invalid) |
| (8 to 15) | 1 x x x | 0 0 | 1 |
| (4 to 7) | 0 1 x x | 0 1 | 1 |
| (2 to 3) | 0 0 1 x | 1 0 | 1 |
| (1) | 0 0 0 1 | 1 1 | 1 |

For $E_1$




$E_1 = \bar{I_0}\bar{I_1}$

For $E_2$




$E_2 = \bar{I_0}I_1 + \bar{I_0}I_2$

$V = I_1 + I_2 + I_3 + I_4$

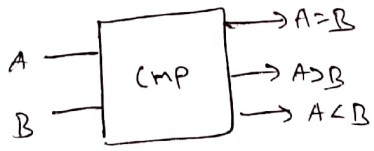
(Cuz it is '0' only if all i/p are 0 i.e., 4 i/p OR gate)

## Magnitude Comparator:

→ It is used to compare two magnitudes (values)

One bit
Two bit
Three bit
n bit

one bit cmp:



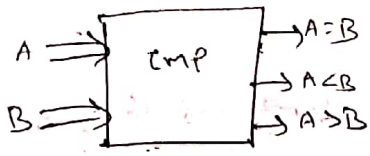

| A B | A=B | A<B | A>B |
|---|---|---|---|
| 0 0 | 1 | 0 | 0 |
| 0 1 | 0 | 1 | 0 |
| 1 0 | 0 | 0 | 1 |
| 1 1 | 1 | 0 | 0 |

$F_{A=B} = A \odot B$

$F_{A<B} = \bar{A}B$

$F_{A>B} = A\bar{B}$

## 2 bit comperator




| A B | A=B | A>B | A<B |
|---|---|---|---|
| 00 00 | 1 | 0 | 0 |
| 0001 | 0 | 0 | 1 |
| 00 10 | 0 | 0 | 1 |
| 00 11 | 0 | 0 | 1 |
| 0100 | 0 | 1 | 0 |
| 0101 | 1 | 0 | 0 |
| 0110 | 0 | 0 | 1 |
| 0111 | 0 | 0 | 1 |
| 1000 | 0 | 1 | 0 |
| 1001 | 0 | 1 | 0 |
| 1010 | 1 | 0 | 0 |
| 1011 | 0 | 0 | 1 |
| 1100 | 0 | 1 | 0 |
| 1101 | 0 | 1 | 0 |
| 1110 | 0 | 1 | 0 |
| 1111 | 1 | 0 | 0 |

Here

one data size = 2

combination size = 4

Max no of combinations = $2^4$

$F_{A=B}$ provides high o/p for $2^2$ combinations

$\therefore$ = 4

$F_{A<B}$ - - - - $\frac{2^4-2^2}{2} = 6$

$F_{A>B}$ - - - - $\frac{2^4-2^2}{2} = 6$

In n bit comperator

one data size = n bit

combination size = $2n$ bits

Max no of combinations = $2^{2n}$

No of A=B combination = $2^n$

No of A<B " = $\frac{2^{2n}-2^n}{2}$

No of A>B " = $\frac{2^{2n}-2^n}{2}$

(2012)

Q No of A>B combinations that provide high o/p in 2 bit comparator is

6

i.e., $\frac{2^{2n}-2^n}{2} = \frac{2^4-2^2}{2} = \frac{16-4}{2} = 6$

Q: No of A<B combinations that provide high o/p in 4 bit comparator is 120

i.e. $\frac{2^{2n}-2^n}{2} = \frac{2^8-2^4}{2} = \frac{256-16}{2} = \frac{240}{2} = 120$

## 5. Sequential Circuits

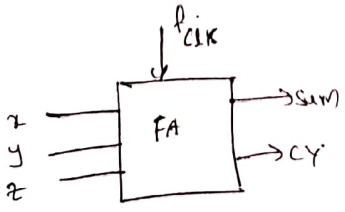
→ All sequential circuits are clocked circuits and frequency of the clock pulse computes the o/p arrival time.

→ Generally universal gates are used to design a sequential circuits.

→ In the given sequential circuit if the clock i/p signal is '0' then no changes are present at the o/p irrespective of i/p data.

→ To operate the ckt for sequential circuit reliable operation, the max clock frequency to be applied is less than $1/t_{Pd}$ where $t_{Pd}$ is ckt propagation delay

Consider below FA



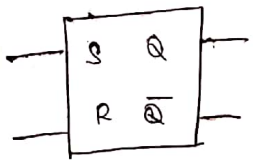

$T_{sum} = 1ns$

$T_{cy} = 1ns$

| $f_{CLK}$ | T (ns) |
|---|---|
| 100 MHz | 10 |
| 200 MHz | 5 |
| 500 MHz | 2 |
| 900 MHz | 1.1 |
| 1 GHz | 1 |
| 2 GHz | 0.5 (X) not permitted because it is less than minimum time delay given |

## SR Latch

→ It is the pre-stage for a Flip flop



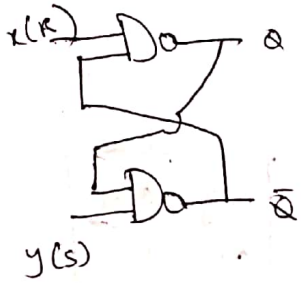

S = set = It sets the 'Q' O/P

R = Reset = It resets the 'Q' O/P

whenever $Q = \bar{Q}$ then that type of i/p is considered as invalid.

→ It contains memory, but no clock.

## SR NAND Latch




| x y | Q Q̄ |
|---|---|
| 0 0 | 1 1 (invalid) |
| 0 1 | 1 0 |
| 1 0 | 0 1 |
| 1 1 | N.C |

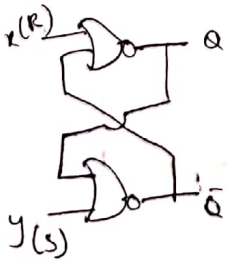
| x y | Q |
|---|---|
| 0 0 | x |
| 0 1 | 1 set |
| 1 0 | 0 Reset |
| 1 1 | NC |

∴ y ⇒ x = R

=>

| S R | Q |
|---|---|
| 0 0 | x |
| 0 1 | 0 |
| 1 0 | 1 |
| 1 1 | NC |

## SR NOR Latch




| x y | Q Q̄ |
|---|---|
| 0 0 | N.C |
| 0 1 | 1 0 |
| 1 0 | 0 1 |
| 1 1 | 0 0 (invalid) |

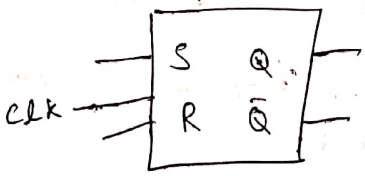
| x y | Q |
|---|---|
| 0 0 | NC |
| 0 1 | 1 set |
| 1 0 | 0 Reset |
| 1 1 | x |

∴ y = S, x = R

| S R | Q |
|---|---|
| 0 0 | NC |
| 0 1 | 0 |
| 1 0 | 1 |
| 1 1 | x |

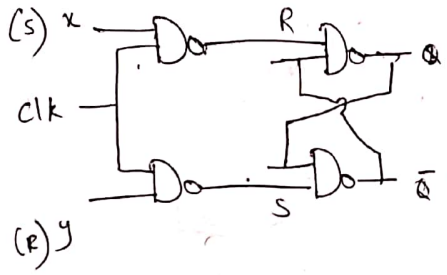
# SR Flip flop

→ When clock pulse is connected to SR latch then it acts as SR Flip Flop




## ~~SR Flip flop~~

~~→ when clock pulse is connected~~ 1



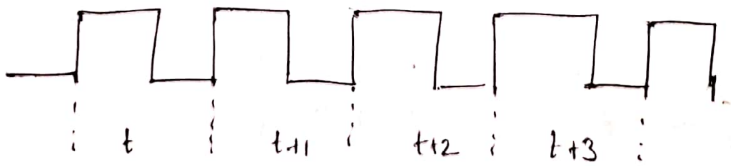

| x y | Q $\bar{Q}$ |
|---|---|
| 0 0 | NC |
| 0 1 | 0 1 |
| 1 0 | 1 0 |
| 1 1 | 1 1 (x) invalid |

} Here Clk is not zero

| x y | Q |
|---|---|
| 0 0 | NC |
| 0 1 | 0 reset |
| 1 0 | 1 set |
| 1 1 | x |

| S R | Q |
|---|---|
| 0 0 | NC |
| 0 1 | 0 |
| 1 0 | 1 |
| 1 1 | x |

Clk



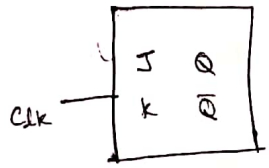

Output (Q) $Q_t$ $Q_{t+1}$ $Q_{t+2}$ $Q_{t+3}$

$Q_t$ = Present State (PS)

$Q_{t+1}$ = Next state (NS)

| S R | $Q_{t+1}$ |
|---|---|
| 0 0 | $Q_t$ (NC) |
| 0 1 | 0 |
| 1 0 | 1 |
| 1 1 | x |

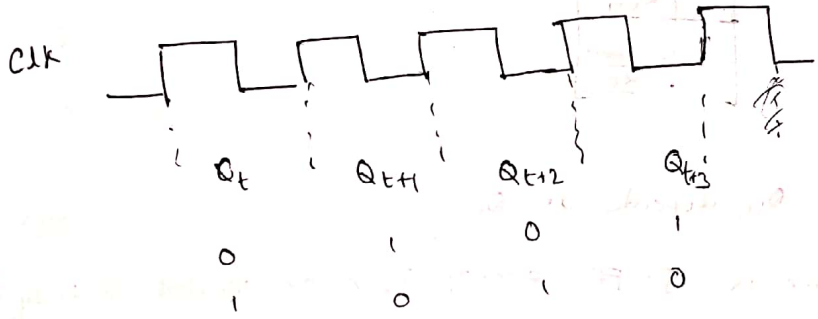
# JK Flipflop




| JK | $Q_{t+1}$ | |
|---|---|---|
| 00 | $Q_t$ | Like SR FF |
| 01 | 0 | |
| 10 | 1 | |
| 11 | $\bar{Q}_t$ (Toggle) | |

Toggle: It is the process of complementing the o/p of a FF for each new clock

→ Toggle is used in counter design.



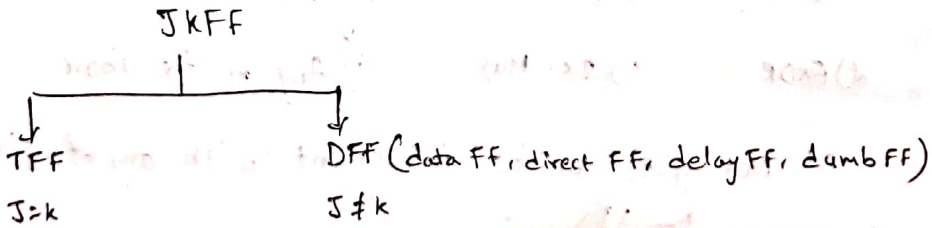

$Q_{t+1} = \bar{Q}_t$

3 bit UP Counter

| $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

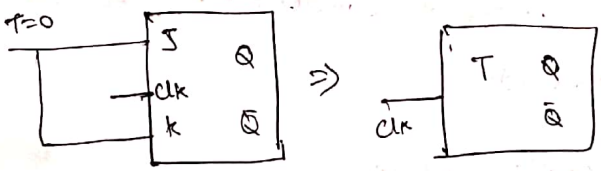
## Single input flip flops

→ ~~Excition~~ Excitation table is easy.



| | JK | $Q_{t+1}$ |
|---|---|---|
| TFF | 00 | $Q_t$ |
| | 11 | $\bar{Q}_t$ |
| DFF | 01 | 0 |
| | 10 | 1 |

## TFF using JKFF



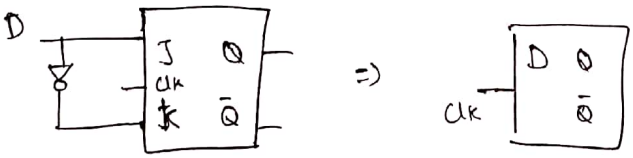

| T | Jk | $Q_{t+1}$ |
|---|---|---|
| 0 | 00 | $Q_t$ |
| 1 | 11 | $\bar{Q}_t$ |

| T | $Q_{t+1}$ |
|---|---|
| 0 | $Q_t$ |
| 1 | $\bar{Q}_t$ |

→ In TFF $Q_{t+1}$ depends on $Q_t$

→ It is same as JK FF except J & K are shorted so they are equal always (J=K)

## DFF using JKFF




| D | Jk | $Q_{t+1}$ |
|---|---|---|
| 0 | 01 | 0 |
| 1 | 10 | 1 |

| D | $Q_{t+1}$ |
|---|---|
| 0 | 0 |
| 1 | 1 |

→ Here J ≠ k.

Q: To implement DFF using JKFF we require

a) NOT gate b) ~~NAD~~ NAND e) NOR

d) EXOR e) 2x1 Mux f) Any of the above ✓

∴ Not gate can be implement with any of given 5 gates

NOT with XOR: 

# Flip Flop Tables

→ Each FF table can be divided in 3 ways:

(i) General table
(ii) Characteristic table
(iii) Excitation table

→ General table tells FF's nature. (Tables that are drawn so far are general tables)

→ Charateristic table tells the relations blw $Q_t$ & $Q_{t+1}$

→ Excitation table is used to design any synchronous sequential circuit.

$Q_t$ : Present state (expected)

$Q_{t+1}$ : Next state (final)

## Characteristic tables:

### DFF:

| D | $Q_t$ | $Q_{t+1}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$Q_{t+1} = D\bar{Q}_t + DQ_t$

$Q_{t+1} = D$ (Characteristic equation)

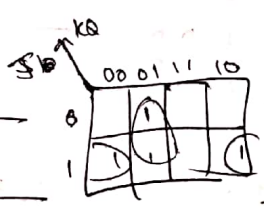
### T FF

| | T | $Q_t$ | $Q_{t+1}$ |
|---|---|---|---|
| NC | 0 | 0 | 0 |
| | 0 | 1 | 1 |
| Toggle | 1 | 0 | 1 |
| | 1 | 1 | 0 |

$Q_{t+1} = T \oplus Q_t$ (Characteristic eq of TFF)

### JK FF

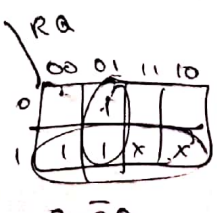
| | J | K | $Q_t$ | $Q_{t+1}$ |
|---|---|---|---|---|
| NC | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| | 1 | 0 | 1 | 1 |
| Toggle | 1 | 1 | 0 | 1 |
| | 1 | 1 | 1 | 0 |




$Q_{t+1} = J\bar{Q} + \bar{K}Q$

### SR FF

| S | R | $Q_t$ | $Q_{t+1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | x |
| 1 | 1 | 1 | x |




$S + \bar{R}Q$

## Excitation tables:

→ First write our target states ($Q_t$ & $Q_{t+1}$)

→ Then define the i/p for the available Flip Flop

### TFF

General table

| T | $Q_{t+1}$ |
|---|---|
| 0 | $Q_t$ (NC) |
| 1 | $\bar{Q}_t$ |

Excitation table

| $Q_t$ | $Q_{t+1}$ | T |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

### DFF

General table

| D | $Q_{t+1}$ |
|---|---|
| 0 | 0 |
| 1 | 1 |

| $Q_t$ | $Q_{t+1}$ | D |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

### JKFF

| | JK | $Q_{t+1}$ |
|---|---|---|
| T | 00 | NC |
| | 11 | Toggle |
| D | 01 | 0 |
| | 10 | 1 |

| $Q_t$ | $Q_{t+1}$ | JK | |
|---|---|---|---|
| 0 | 0 | 00 | No change |
| | | 01 | Direct '0' o/p |
| | | 0X | |
| 0 | 1 | 11 | Toggle |
| | | 10 | Direct '1' o/p |
| | | 1X | |
| 1 | 0 | 11 | Toggle |
| | | 01 | Direct '0' o/p |
| | | X1 | |
| 1 | 1 | 00 | No change |
| | | 10 | Direct '0' o/p |
| | | X0 | |

→ Here in building exciting table each $Q_tQ_{t+1}$ is checked for possibility in DFF & TFF

Eg: for 00 we can see it as No change (or) Direct '0' o/p

## SRFF

| SR | $Q_{t+1}$ |
|---|---|
| 00 | NC |
| 01 | 0 |
| 10 | 1 |
| 11 | x |

| $Q_t$ | $Q_{t+1}$ | S R | |
|---|---|---|---|
| 0 | 0 | 00 | — No change. |
| | | 01 | — '0': reset by R |
| | | 0x | |
| 0 | 1 | 10 | — '1': set by S |
| 1 | 0 | 01 | — '0': reset by R |
| 1 | 1 | 00 | — No change |
| | | 10 | — '1': set by S |
| | | x0 | |

## Counter & It's design

→ Counter is used to count the no of states.

→ Each state is available for one clock cycle time.

→ n bit counter produces max $2^n$ states.

→ If n bit counter is used to count all $2^n$ states then it is binary counter otherwise it is mod counter.

→ Mod counter is used to count the target no of states only.

→ Mod k counter is used to count k no of states.

→ In real time applications mod counter is useful than binary counter, but design cost of mod counter is more expensive than design cost of binary counter.

→ There are two methods to design a counter:

(i) Synchronous counter

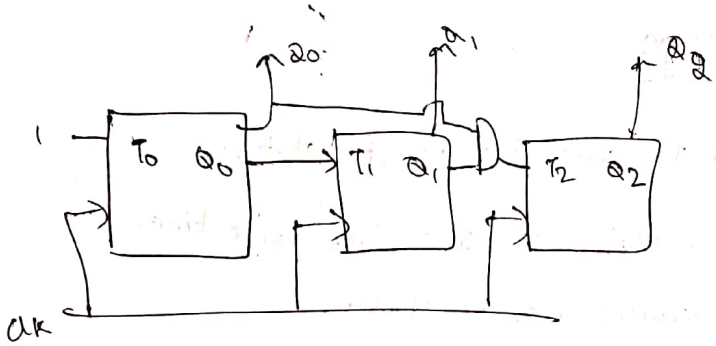
(ii) Asynchronous counter (Ripple counter)

## Design of Synchronous Counter:

→ All flip flops' clock i/ps are connected to main clock generator and flip flop i/ps are defined with excitation table.

## 3 bit upcounter

| | $Q_2Q_1Q_0$ | $T_2$ | $T_1$ | $T_0$ |
|---|---|---|---|---|
| $Q_t$ | 0 0 0 | 0 | 0 | 1 |
| $Q_{t+1}$ | 0 0 1 | 0 | 1 | 1 |
| $Q_{t+2}$ | 0 1 0 | 0 | 0 | 1 |
| $Q_{t+3}$ | 0 1 1 | 1 | 1 | 1 |
| : | 1 0 0 | 0 | 0 | 1 |
| : | 1 0 1 | 0 | 1 | 1 |
| : | 1 1 0 | 0 | 0 | 1 |
| $Q_{t+7}$ | 1 1 1 | 1 | 1 | 1 |

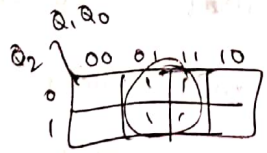



O/P : $Q_2Q_1Q_0$

$T_2(Q_2Q_1Q_0) = \bar{Q}_2 Q_1 Q_0 + Q_2 Q_1 Q_0$

$= Q_1 Q_0$

for $T_1$ :



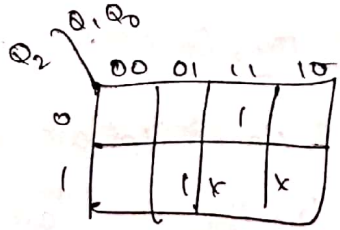

$T_1(Q_2Q_1Q_0) = Q_0$

for ($T_0$)

$T_0(Q_2Q_1Q_0) = 1$

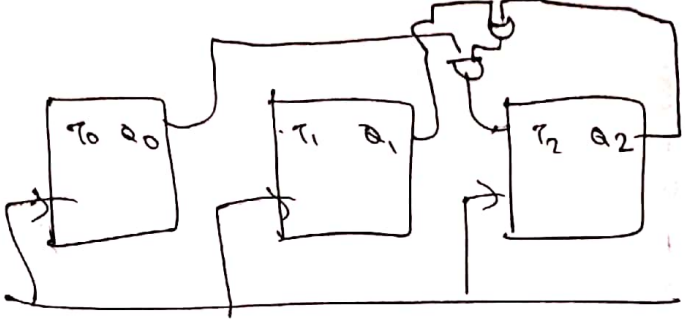
## Mod 6 up counter:

| | $Q_2Q_1Q_0$ | $T_2$ | $T_1$ | $T_0$ |
|---|---|---|---|---|
| ↑ | 0 0 0 | 0 | 0 | 1 |
| | 0 0 1 | 0 | 1 | 1 |
| | 0 1 0 | 0 | 0 | 1 |
| | 0 1 1 | 1 | 1 | 1 |
| | 1 0 0 | 0 | 0 | 1 |
| | 1 0 1 | 1 | 1 | 1 |
| | 1 1 0 | 0 | 0 | x |
| | 1 1 1 | x | x | x |




$T_2 = Q_1Q_0 + Q_2Q_0$

$= Q_0(Q_1 + Q_2)$




→ Here $T_2$ ~~clr~~ & alone req 2 gate which makes mod counter design expensive

Eg: Design a counter for the states of 0,3,6,2,5, ~~and rep~~

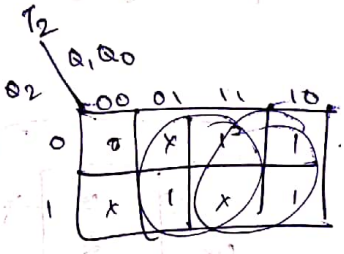
String: - 0,3,6,2,5

No state is repeated

Highest digit = 6 (110)

∴ No of FF's req = 3

| | $Q_2$ | $Q_1$ | $Q_0$ | $T_2$ | $T_1$ | $T_0$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 1 | 1 |
| 5 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | x | x | x |
| 4 | 1 | 0 | 0 | x | x | x |
| 7 | 1 | 1 | 1 | x | x | x |



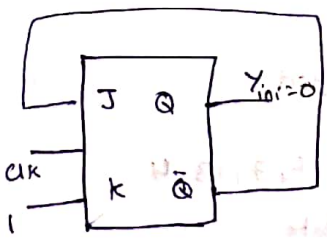

$T_2 = Q_1 + Q_0$

Sly derive expression for $T_1$ & $T_2$

08/09/19

Q) In the below diagrams initial o/p at Y is '0'. what is output signal at 'Y' for next 3 clocks?

1)




a) 1,1,1

b) 0,0,0

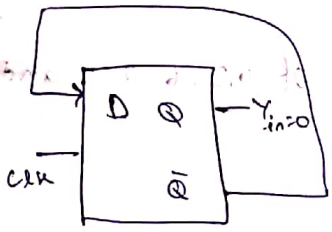
c) 1,0,1

d) 0,1,0

| clk | J | K | Q | $\bar{Q}$ |
|---|---|---|---|---|
| | | | 0 | 1 |
| ↑ | 1 | 1 | 1 | 0 |
| ↑ | 0 | 1 | 0 | 1 |
| ↑ | 1 | 1 | 1 | 0 |

∴ 1, 0, 1

2)

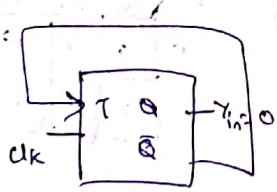



a) 0, 0, 0
b) 1, 1, 1
c) 1, 0, 1
d) 0, 1, 0

| clk | D | Q | $\bar{Q}$ |
|---|---|---|---|
| | | 0 | 1 |
| ↑ | 1 | 1 | 0 |
| ↑ | 0 | 0 | 1 |
| ↑ | 1 | 1 | 0 |

∴ 1, 0, 1

3)




a) 0, 0, 0
b) 1, 1, 1
c) 1, 0, 1
d) 0, 1, 0

| clk | T | Q | $\bar{Q}$ |
|---|---|---|---|
| | | 0 | 1 |
| ↑ | 1 | 1 | 0 |
| ↑ | 0 | 1 | 0 |
| ↑ | 0 | 1 | 0 |

∴ 1, 1, 1

**Q**: Min no of FFs req for designing a counter for generating the states 2, 5, 0, 9, 3, 1, 12, 6, 7, 13, 4 and repeats is 4

~~Max value = 14 (1110)~~

~~ie, 4 FFs are needed~~

Given string 2, 5, 0, 9, 3, 1, 12, 6, 7, 13, 4

There is no repeated state

Highest state = $13_{10} = (1101)_2$

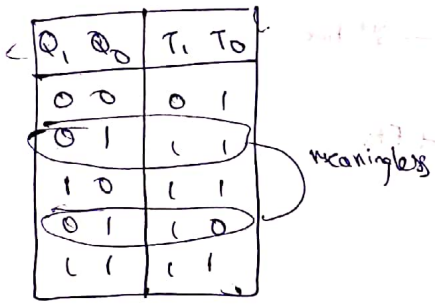
∴ 4 FFs are needed

Q: Minimum no of FFs req for producing ~~0,0,1,1~~ 0,1,2,1,3 and repeats is 3

Sol:
Given string is 0,1,2,1,3

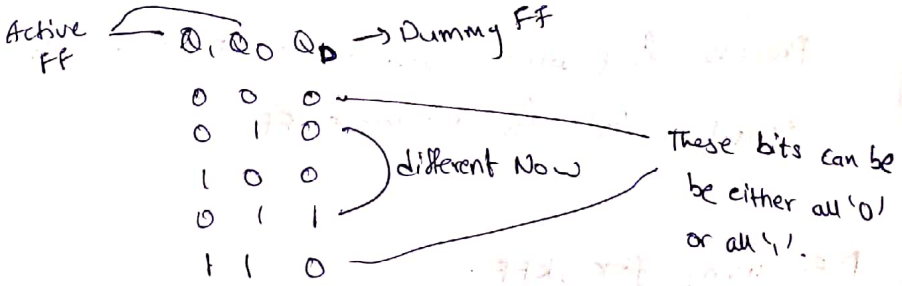
state '1' is repeated, hence this design requires additional FFs known as dummy FFs

Note:
Before writing the excitation table all states should be distinct.




∴ To distinguish same states we use a dummy flipflop

So Consider the excitation table as




∴ We need 3 FFs

## Problems

(2015)

6) Given 0,0,1,1,2,2,3,3

Each state repeated for 2 times

Max digit = 3 = $(11)_2$

| $Q_0$ | $Q_1$ | $Q_D$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

∴ We need 3 FFs

2 active FFs
1 dummy FF

2016

1) Given string 0, 1, 0, 2, 0, 3

Max value = $3 = (11)_2$

∴ 2 active FFs needed

0 is repeated for 3 time

So we need two Dummy FFs

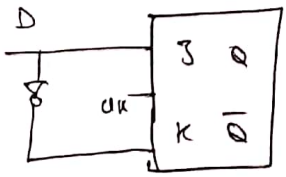
| $Q_1$ | $Q_0$ | $Q_{D_1}$ | $Q_{D_2}$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | — 1st time |
| 0 | 1 | 0 | 0 | |
| 0 | 0 | 0 | 1 | — 2nd time |
| 1 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | — 3rd time |
| 1 | 1 | 0 | 0 | |

∴ So we need 4 FFs

## Flip Flop Conversions:

→ Any target FF can be implemented with available FF. In this Process first write characteristic table for the target FF then excitation table for available FF.

### DFF using ~~JKK~~ JKFF:

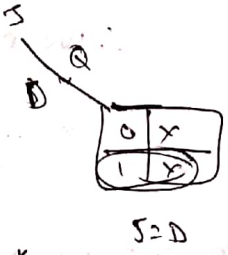



$J = D$

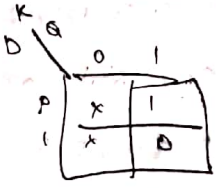
$K = \bar{D}$

$J = Q\bar{K}$

| D | Q | $Q_{t+1}$ | J | K |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | x |
| 0 | 1 | 0 | x | 1 |
| 1 | 0 | 1 | 1 | x |
| 1 | 1 | 1 | x | 0 |




$J = D$



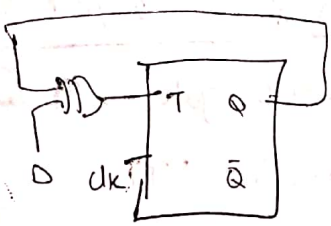

$K = \bar{D}$

## DFF using TFF :

| D | $Q_t$ | $Q_{t+1}$ | T |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

$T = \bar{D}Q + D\bar{Q}$

$T = D \oplus Q$

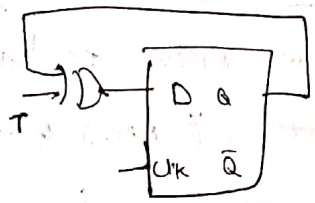



## TFF using DFF

| T | $Q_t$ | $Q_{t+1}$ | D |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

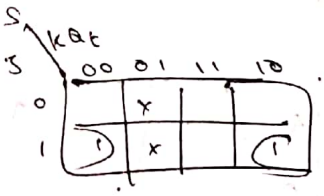
$D = \bar{T}Q_t + T\bar{Q}_t$

$= T \oplus Q_t$

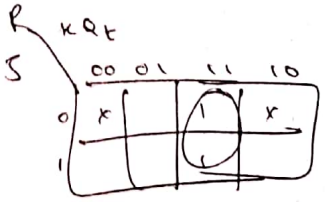



## JkFF using SRFF

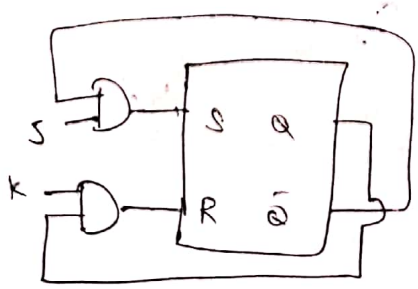
| | Jk | $Q_t$ | $Q_{t+1}$ | SR |
|---|---|---|---|---|
| NC | 00 | 0 | 0 | 0 x |
| | 00 | 1 | 1 | x 0 |
| 0 | 01 | 0 | 0 | 0 x |
| | 01 | 1 | 0 | 0 1 |
| 1 | 10 | 0 | 1 | 1 0 |
| | 10 | 1 | 1 | x 0 |
| toggle | 11 | 0 | 1 | 1 0 |
| | 11 | 1 | 0 | 0 1 |

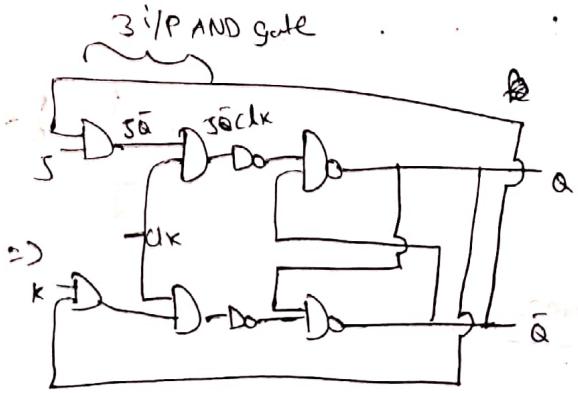


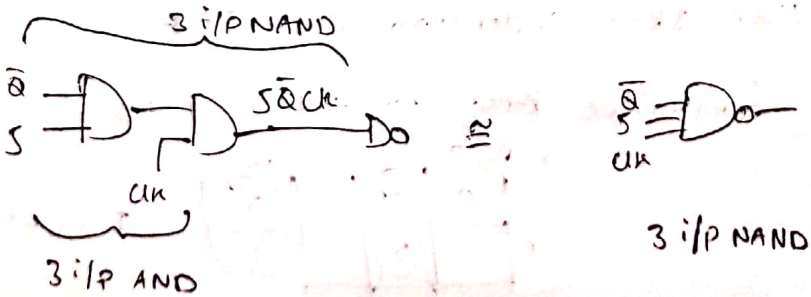

$S = J\bar{Q}$



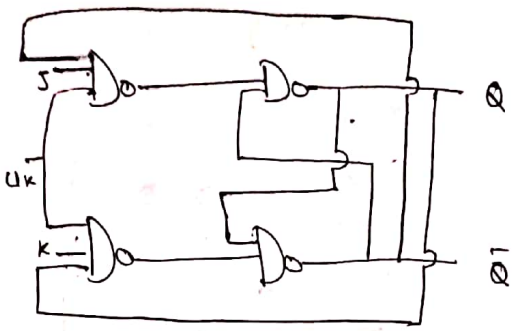

$R = kQ$



⇒



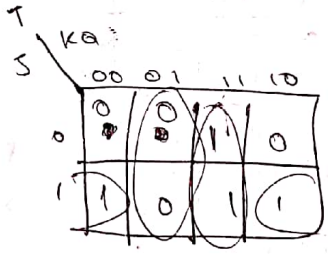

From

⇒ ~~Fro~~ above diagram we can conclude

* JKFF requires two 3 i/p NAND & two 2 i/p NAND gates
* If we remove feedbacks for 3 i/p NAND gates and replace ~~Make~~ with ~~then~~ 2 i/p NAND gates then it turns into SRFF.
* SRFF requires four 2 i/p NAND gates.

## JKFF using TFF

| J K | $Q_t$ | $Q_{t+1}$ | T | |
|---|---|---|---|---|
| 0 0 | 0 | 0 | 0 | NC |
| 0 0 | 1 | 1 | 0 | |
| 0 1 | 0 | 0 | 0 | 0 |
| 0 1 | 1 | 0 | 1 | |
| 1 0 | 0 | 1 | 1 | 1 |
| 1 0 | 1 | 1 | 0 | |
| 1 1 | 0 | 1 | 1 | Toggle |
| 1 1 | 1 | 0 | 1 | |




$T = KQ + J\bar{Q}$

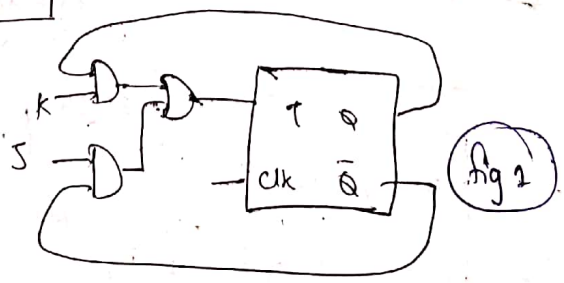
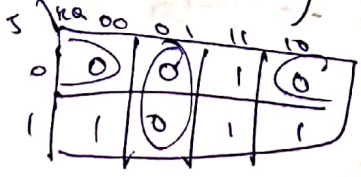



fig 1

## Problems

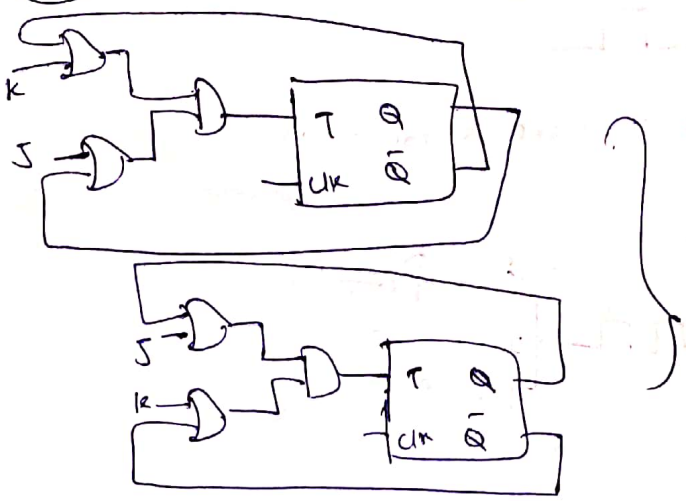
3) Given JKFF implemented with TFF

from above k-map for finding T is




$T = (K + \bar{Q})(J + Q)$

Here by finding SOP expression none of options matches to (fig 1), So we find P.O.S




## Flip Flop Triggering:

→ Triggering is also known as clocking.

→ Triggering is the process of applying high signal on clock terminal at our desired time interval.

→ Different trigger techniques are

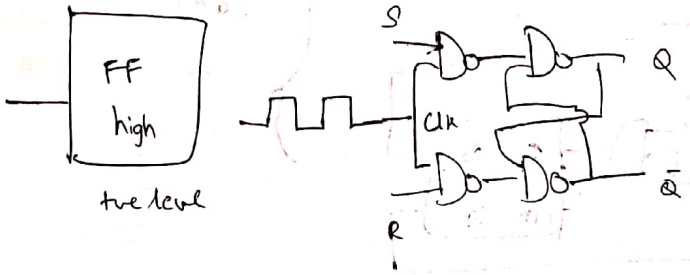
(i) +ve level
(ii) -ve level
} level triggered FF

(iii) +ve edge
(iv) -ve edge
} edge triggerd FF

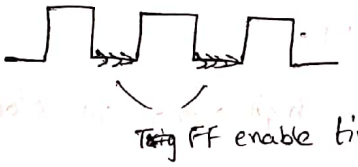
→ Race Around Condition (RAC) occurs in level triggered flip flops. Hence edge triggered flip flop became familiar in real time applications, but schmitt trigger (pulse trainer) is required for designing edge triggered flip flop for providing the high signal to clock terminal at user's target time interval.

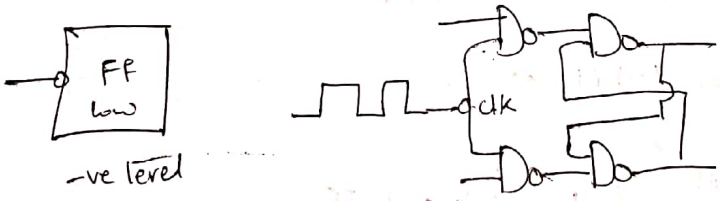
## +ve level trigger

→ The FF is enabled at high signal times

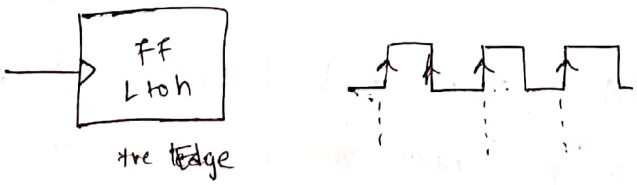



## -ve level trigger

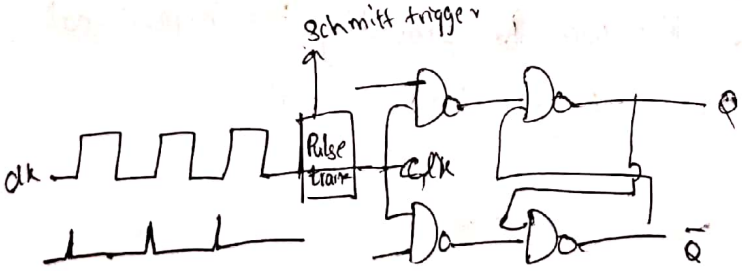


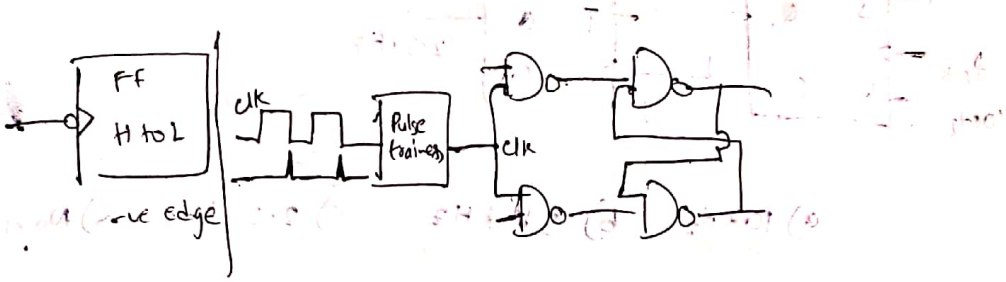

## +ve edge trigger




→ FF is enabled exactly once for each cycle for short time.



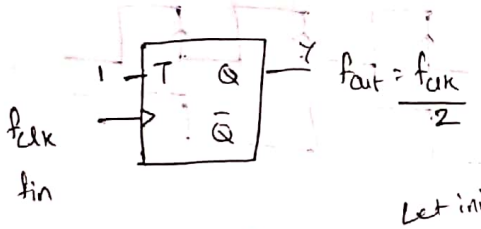
-ve edge trigger



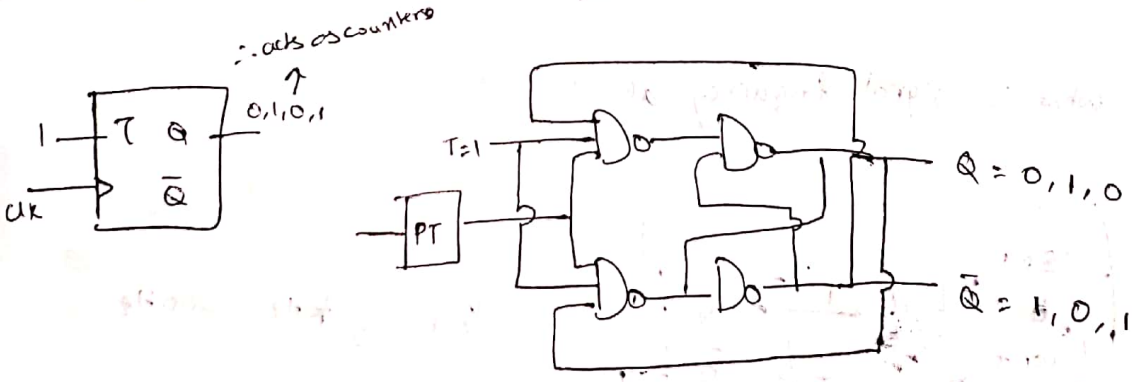

→FF is enabled exactly once for each cycle for short time

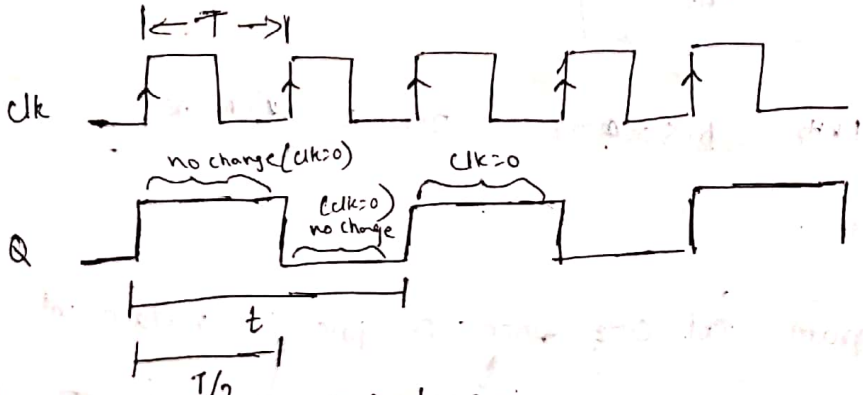
## TFF:

It is also known as frequency divider and mod 2 counter when T=1




Let initial value of Q be 0

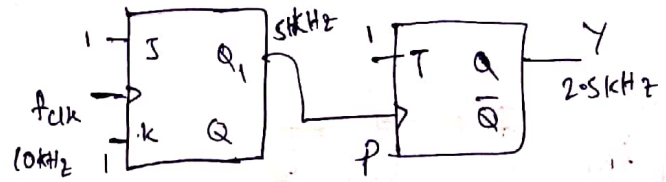


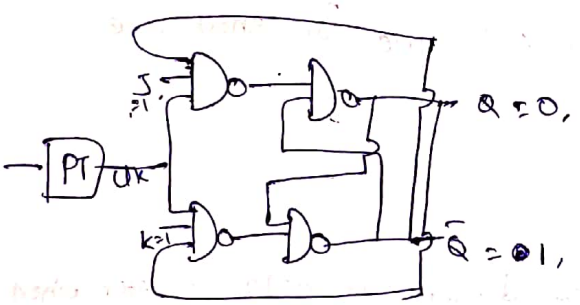

$\Rightarrow t = 2T$

$\Rightarrow f_{out} = f_{in}/2$

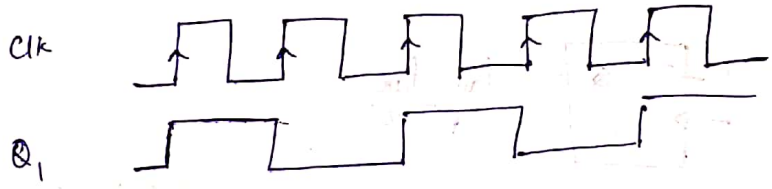
Q: In below diagram what is the frequency at Y?



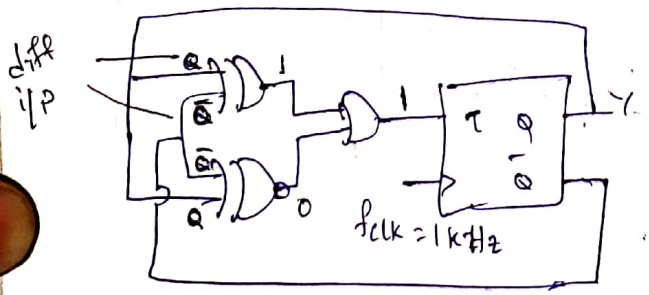

a) 10kHz  b) 5kHz  c) 2·5 kHz  d) None




| J | K | $Q_t$ | $\bar{Q}_t$ | $Q_{t+1}$ | $\bar{Q}_{t+1}$ |
|---|---|---|---|---|---|
| | | 0 | 1 | | |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | | |




Q: what is signal frequency at Y




$f_{out} = \frac{1}{2} \times 1kHz = 500Hz$

a) 1kHz  b) 500Hz  c) 250Hz  d) None

Note:

In above diagram $f_{out} = 0Hz$ when OR gate is replace with AND gate. Because the AND gate O/P is always 0 for T and TFF produces no change for '0' i/p. Thus the O/P is a straight line (high or low).

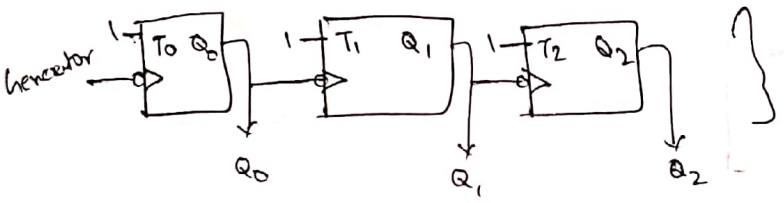
## Design of Ripple Counters (Asynchronous Counter):

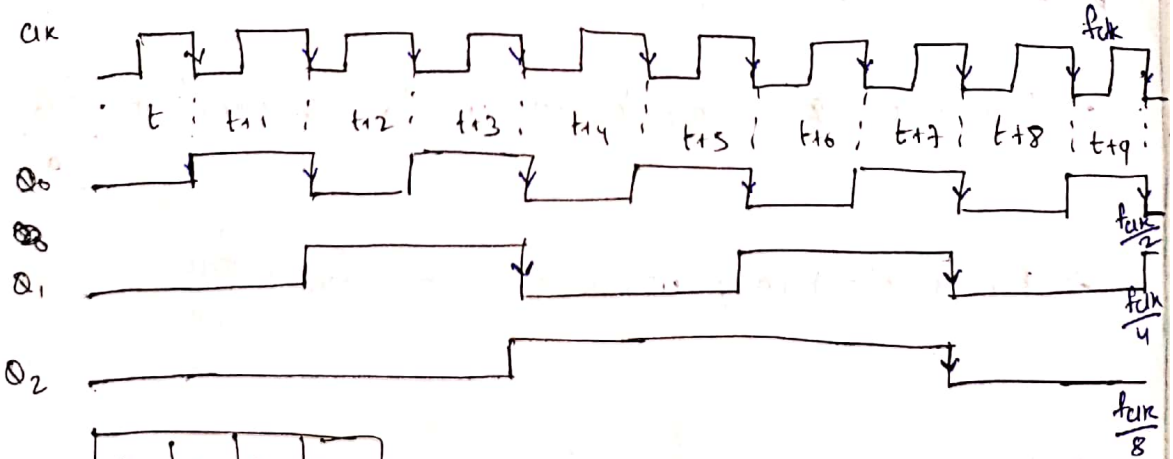
→ Generally TFF are taken and all T i/ps are connected to high signal always, excitation table is not required.

→ Main clock generator is connected to LSB FF only and remainining all clocks are connected in ripple format.

→ Its design cost is cheaper. It is especially used for frequency division application.

### 3 bit up Counter



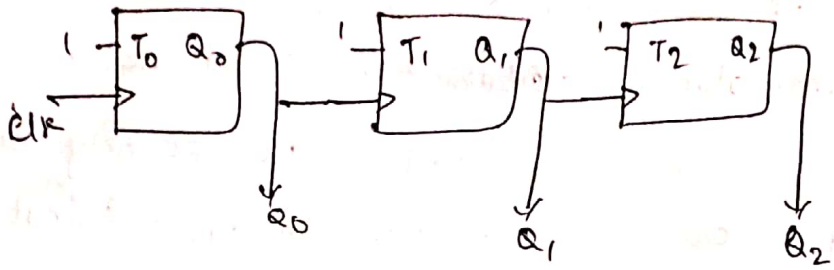

| Clk | $Q_2$ | $Q_1$ | $Q_0$ | |
|---|---|---|---|---|
| t | 0 | 0 | 0 | 0 |
| t+1 | 0 | 0 | 1 | 1 |
| t+2 | 0 | 1 | 0 | 2 |
| t+3 | 0 | 1 | 1 | 3 |
| t+4 | 1 | 0 | 0 | 4 |
| t+5 | 1 | 0 | 1 | 5 |
| t+6 | 1 | 1 | 0 | 6 |
| t+7 | 1 | 1 | 1 | 7 |
| t+8 | 0 | 0 | 0 | 00 |
| t+9 | 0 | 0 | 1 | 01 |

→ The circled ones represent where toggle occured. (i.e, high to low).

→ Once a toggle is occured we change the bit value (inverse) and continue same value until next toggle occurs

The above 2 points help in designing the table without drawing all waves. A shortcut method.

## 3 bit down counter




| clk | $Q_2$ $Q_1$ $Q_0$ | |
|---|---|---|
| t | 0 0 0 | 0 |
| t+1 | 1 1 1 | 7 |
| t+2 | 1 1 0 | 6 |
| t+3 | 1 0 1 | 5 |
| t+4 | 1 0 0 | 4 |
| t+5 | 0 1 1 | 3 |
| t+6 | 0 1 0 | 2 |
| t+7 | 0 0 1 | 1 |
| t+8 | 0 0 0 | 0 |
| t+9 | 1 1 1 | 7 |

**Note**

TFF
+ve edge – down counter

-ve edge TFF – up counter

~~Q: In a 4 bit binary~~

→ For above 3 bit binary up/down counter, $f_{out} = \frac{f_{clk}}{2^3}$

→ For n bit binary up/down counter, $f_{out} = \frac{f_{clk}}{2^n}$

→ For mod k counter $f_{out} = \frac{f_{clk}}{k}$

Q: In a 4 bit binary counter the present state is 0110, what is o/p after 18 clocks

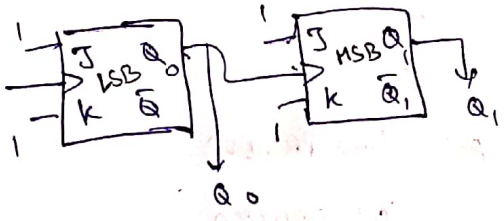
a) 0110 b) 0111 c) 1111 d) 1000 ✓

→ For every 16 clocks ($2^4$) value will be repeated

∴ 18 % 16 = 2 ⇒ $0110 + (10)_2 = 1000$

2011

Q: In the below diagram, initial o/p at $Q_1, Q_0$ is '00' what is o/p for next 4 clocks?

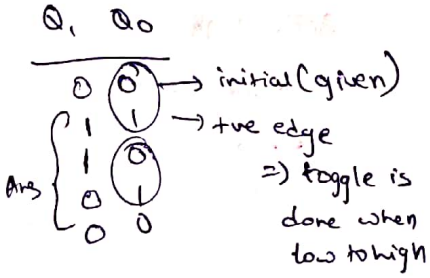



a) 00, 01, 10, 11

b) 11, 10, 01, 00

c) 10, 11, 01, 00

d) None




In the above diagram J = K = 1 i.e., TFF with T = 1

∴ The ~~the~~ diagram represents 2 bit binary down counter

~~i.e. 00, 01, 10, 11~~

=> 0, 3, 2, 1, 0

11, 10, 01, 00

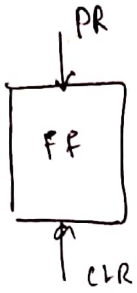
## Design of asynchronous mod counters :

→ This design requires 2 special inputs of a flip flop known as preset (pr) and clear (clr) (asynchronous inputs).

→ These are also known as (pr & clr) asynchronous i/ps.

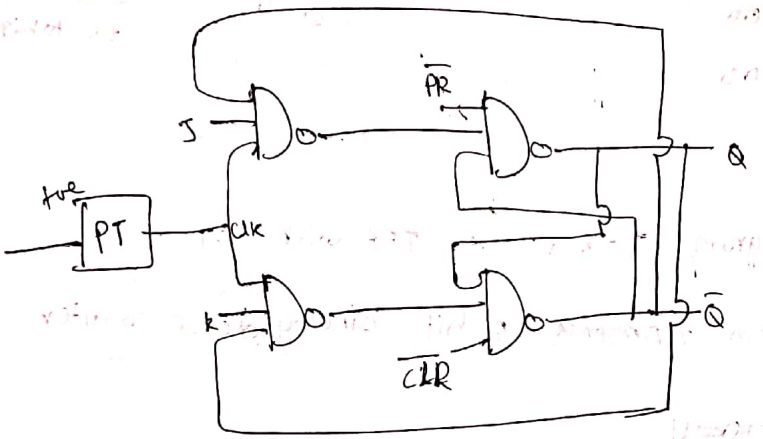
★ → preset is used to design down counter and clear is used to design up counter.

→ During normal operations these i/ps should be in disabled condition because if preset is enabled o/p filpflop will be forced to high and if clear is enabled the o/p the flipflop will be forced to low immediately regardless of flipflop data i/ps & clock

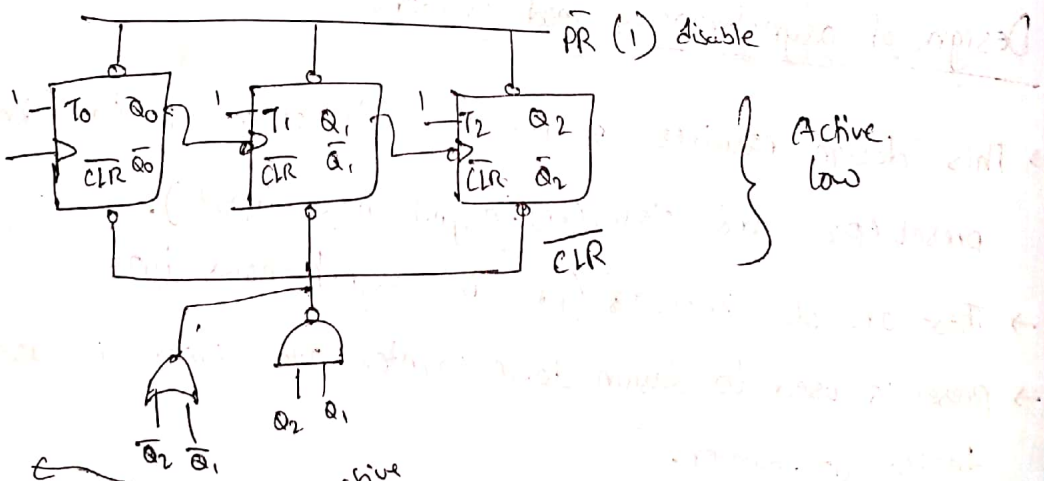



Active high
'0'-disabled
'1'-enabled


Active low PR, CLR
'0'-enabled
'1'-disabled




## Mod 6 upcounter

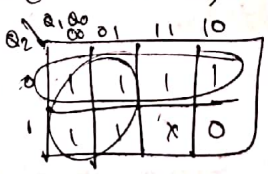



active high ← | → active high

| $Q_2Q_1Q_0$ | $\overline{CLR}$ | CLR |
|---|---|---|
| 000 | 1 | 0 |
| 001 | 1 | 0 |
| 010 | 1 | 0 |
| 011 | 1 | 0 |
| 100 | 1 | 0 |
| 101 | 1 | 0 |
| 110 | 0 | 1 |
| 111 | X | X |

$\overline{CLR}(Q_2Q_1Q_0) = \Sigma m(0,1,2,3,4,5) + d(7)$

$= \overline{Q_2Q_1}$

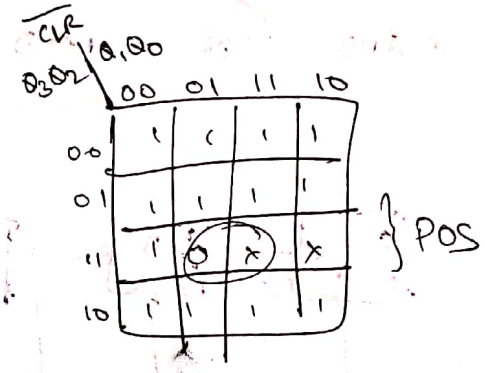



Silly

$CLR(Q_2Q_1Q_0) = Q_2Q_1$

## Mod 13 up counter

| $Q_3\ Q_2\ Q_1\ Q_0$ | $\overline{CLR}$ (active low) | CLR (active high) |
|---|---|---|
| 0 0 0 0 | 1 | 0 |
| 0 0 0 1 | 1 | 0 |
| ⋮ | ⋮ | ⋮ |
| 1 0 1 0 | 1 | 0 |
| 1 0 1 1 | 1 | 0 |
| 1 1 0 0 | 1 | 0 |
| 1 1 0 1 | 0 | 1 |
| 1 1 1 0 | x | x |
| 1 1 1 1 | x | x |




$$\overline{CLR} = \overline{Q_3} + \overline{Q_2} + \overline{Q_0}$$

$$= \overline{Q_3 Q_2 Q_0}$$


Sily $CLR = Q_3 Q_2 Q_0$

## Mod 83 upcounter

→ 0 to 82 accept

$83 = (1010011)_2$
$Q_6\ Q_5\ Q_4\ Q_3\ Q_2\ Q_1\ Q_0$

Here we have to stop counting at 83

∴ 83's binary value have 4 1's. So we need four i/p NAND gate for which i/ps are 1's digit place in 83's binary representation

→ No of FFs required = 7

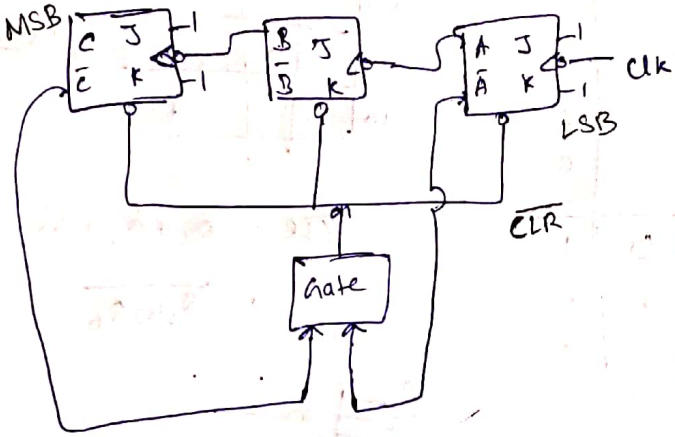
→ Name of gate is NAND (OR can also be used by taking inputs for $\overline{Q_i}$)

→ No of i/ps of NAND(or) OR gate = 4

→ Gate is

Q: In the given below mod 5 counter, the name of the gate is ____

a) AND  b) OR ✓  c) NAND  d) NOR




Since the inputs of gate are complement it is OR gate.

Method II

| C B A | $\overline{CLR}$ | $\bar{C}\bar{A}$ |
|---|---|---|
| 0 0 0 | 1 | 11 |
| 0 0 1 | 1 | 10 |
| 0 1 0 | 1 | 11 |
| 0 1 1 | 1 | 10 |
| 1 0 0 | 1 | 01 |
| 1 0 1 | 0 | 00 |

→ for gate to enable clear it should produce o/p 0.

→ 0 to 0 i.e. OR gate

## Race Around Condition (RAC):

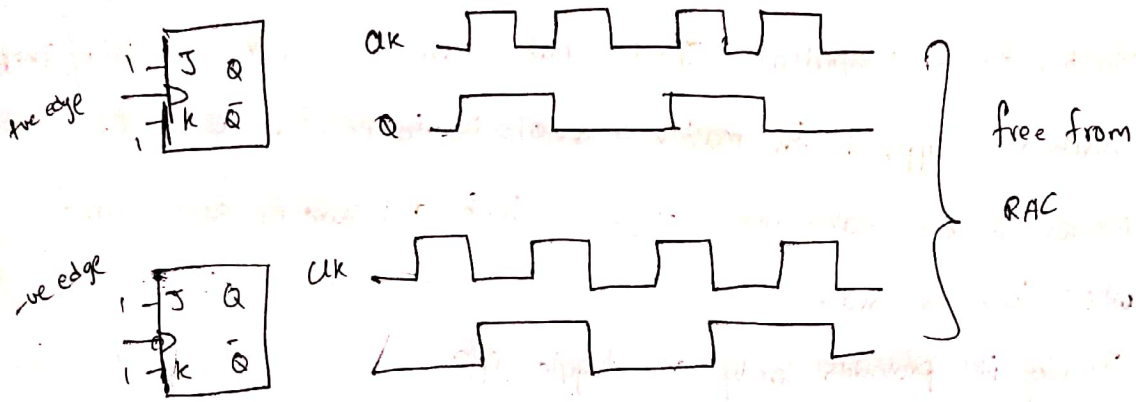
→ When flip flop o/p is toggled more than one time in one clock cycle completion the Race Around Condition occurs.

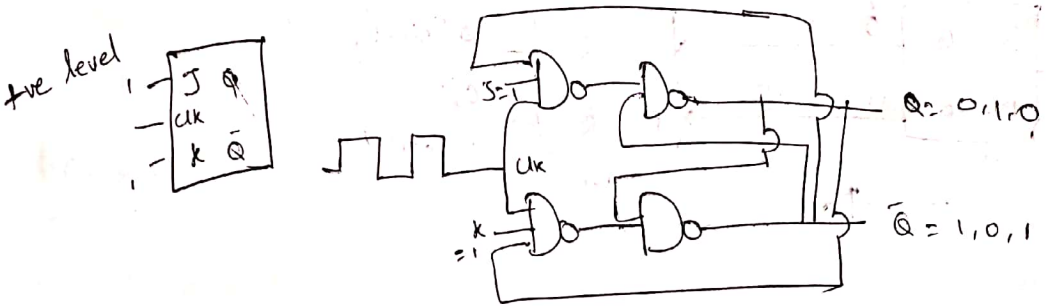
→ SR flip flop and DFF are free from RAC.

→ Edge triggered JkFF and edge triggered TFF are free from RAC.

→ RAC occurs only in level triggered JkFF and level triggered TFF.

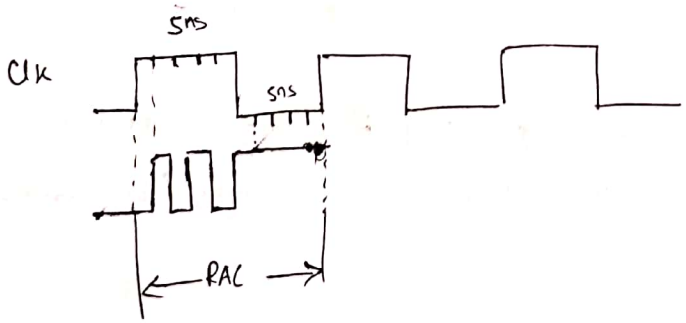
→ To prevent RAC the master slave flipflop is used.




$\Delta t = t_{PD} = 1ns$ (i.e, it takes 1ns to produce Q & $\bar{Q}$ for given i/p)

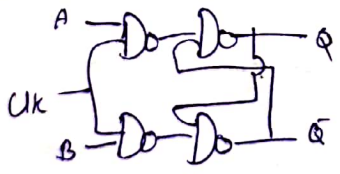
Let frequency of clock be 100 MHz $\Rightarrow$ T = 10ns




In the above figure the o/p is toggled more than once for given on clock cycle.

(2012)

Q: In the below diagram, RAC




This SRFF which is free from RAC

a) does not occur ✓

b) occurs when A=1, B=1, clk=0

c) occurs when A=0, B=0, clk=0

d) occurs when A=1, B=1, clk=1

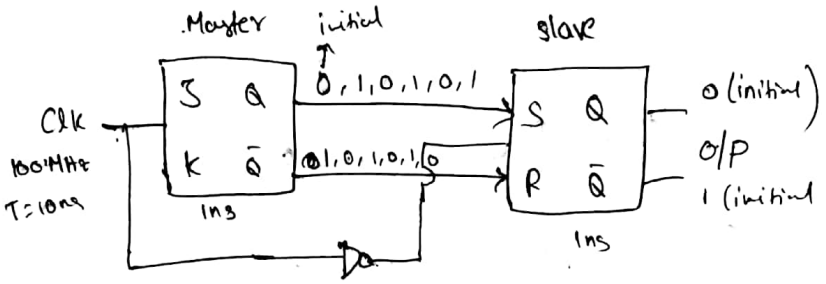
## Master slave JkFF

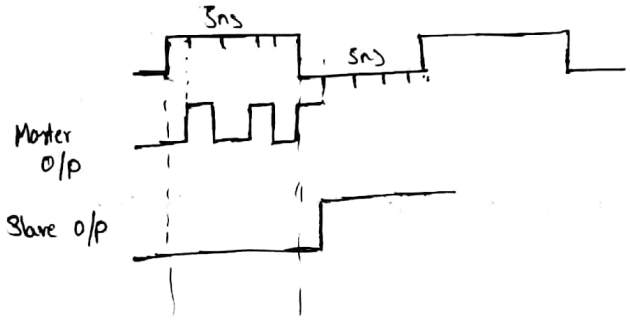
→ Master FF is compulsary JkFF but slave FF can be SRFF/JkFF

→ Inputs are applied for master and o/p is taken from slave FF.

→ Master FF gets enable when clock is high and slave FF gets enable when clock is low.

→ Finally it provides only one toggle o/p.



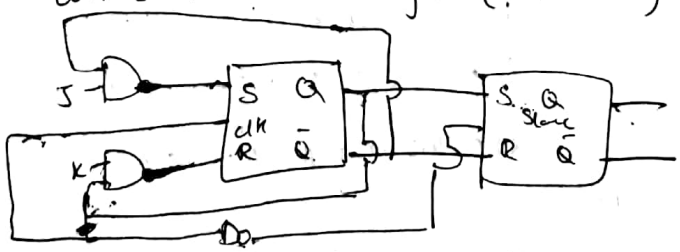




→ The above ~~fig~~ circuit works good only if toggle occured in master in one cycle is odd number of times (Eg: 5 ~~times~~ times in above diagram)

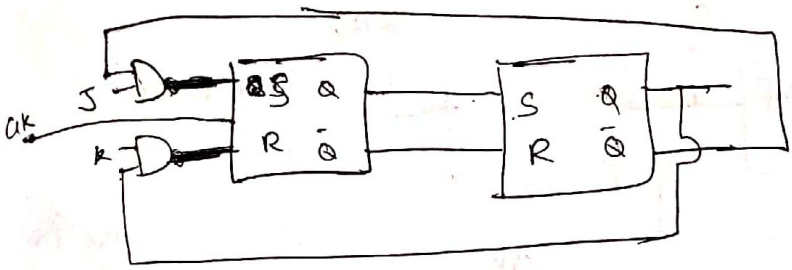
→ If it occurs even number of time the slave o/p is a straight line.

**Modified diagram to toggle the master FF o/p only one time during clock high:**

Corride previous diagram (the above) as shown below



It is modified as




→ The i/p at JK changes only if the feedback to which it is connected change.

→ Here feedback is slave's o/p which changes only once and thus the modified circuit's master's o/p also changes once.

## Registers

→ It is used to save applied data. Hence DFF are used for its design

→ n-bit register design requires n number of DFFs

→ Register is also used to perform binary multiplication and binary division by shifting the data to left and right respectively

→ Data in a register can be shifted in four ways.

(i) Serial in serial out (SISO)
(ii) Serial in parallel out (SIPO)
(iii) Parallel in serial out (PISO)
(iv) Parallel in parallel out (PIPO)

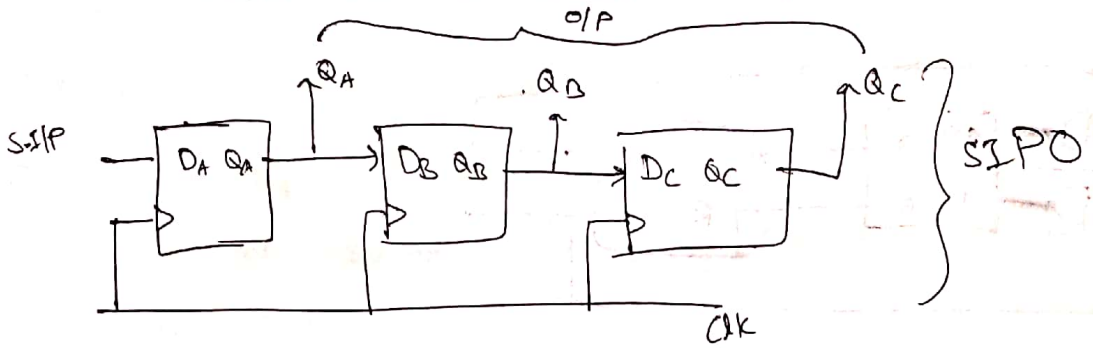
→ In most of the real time application serial In parallel Out technique is used.

$A_3\ A_2\ A_1\ A_0$

Left shift $A_2\ A_1\ A_0\ 0$ (This mulplies data by 2)

Right shift $0\ A_3\ A_2\ A_1$ (This divides data by 2 i.e, Integer division $(5/2=2)$)

Consider below S.I.P.O circuit




$D_A = S.I/P$

$D_B = Q_A$

$D_C = Q_B$

| S-I/P | $D_A$ $D_B$ $D_C$ | Ckt | $Q_A$ $Q_B$ $Q_C$ | |
|---|---|---|---|---|
| | | | 0 0 0 | |
| 1 | 1 0 0 | ↑ | 1 0 0 | to shift bit '1' |
| 1 | 1 1 0 | ↑ | 1 1 0 | |
| 1 | 1 1 1 | ↑ | 1 1 1 | |
| 0 | 0 1 1 | ↑ | 0 1 1 | to shift bit '0' |
| 0 | 0 0 1 | ↑ | 0 0 1 | |
| 0 | 0 0 0 | ↑ | 0 0 0 | |

## Ring Counter (RC)

→ The SIPO register acts as R.C when last FF O/P is connected to first FF input.

→ During initialization, it requires preset technique (only for one of the Flipflops)

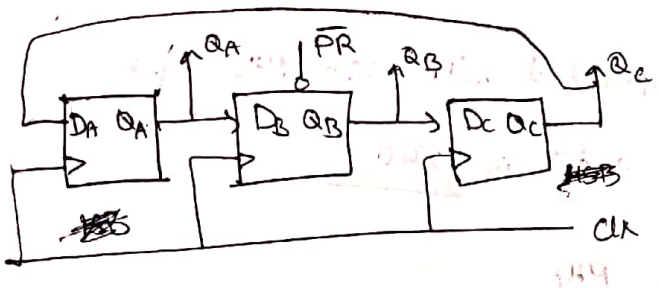
→ In n bit R.C the no of useful states is n and no of unused states is $2^n - n$

→ n bit RC is also known as Mod n counter and fout: $\frac{fin}{n}$.

Eg: A 5 bit RC is operated with 120 MHz clock

① No of unused states is 27.

② fout is 24 MHz.




$D_A = Q_C$

$D_B = Q_A$

$D_C = Q_B$

**Note:**

whatever the size of ring counter, always only one o/p bit is high ~~active~~ like active high o/p decoder.

→ Preset (because initial state requires exactly one '1' bit in $Q_A$ (or) $Q_B$ (or) $Q_C$)

| $D_A$ $D_B$ $D_C$ | Clk | $Q_A$ $Q_B$ $Q_C$ |
|---|---|---|
| initial | | 0 1 0 |
| 0 0 1 | ↑ | 0 0 1 |
| 1 0 0 | ↑ | 1 0 0 |
| 0 1 0 | ↑ | 0 1 0 |

3 useful states out of $2^3$ states

$2^3 - 3 = 5$ states are unused

## Johnson Counter (JC)

→ It is also known as
Twisted R.C
Ringtail Counter and
Moebius counter

→ It does not require preset technique during initialization.

→ n bit JC is also known as mod $2n$ counter and $f_{out}$ is $\frac{f_{in}}{2n}$.

→ In n bit JC no of useful states is $2n$

and no of unused states is $2^n - 2n$

| $D_A$ $D_B$ $D_C$ | Clk | $Q_A$ $Q_B$ $Q_C$ |
|---|---|---|
| initial | | 0 0 0 |
| 1 0 0 | ↑ | 1 0 0 |
| 1 1 0 | ↑ | 1 1 0 |
| 1 1 1 | ↑ | 1 1 1 |
| 0 1 1 | ↑ | 0 1 1 |
| 0 0 1 | ↑ | 0 0 1 |
| 0 0 0 | ↑ | 0 0 0 |

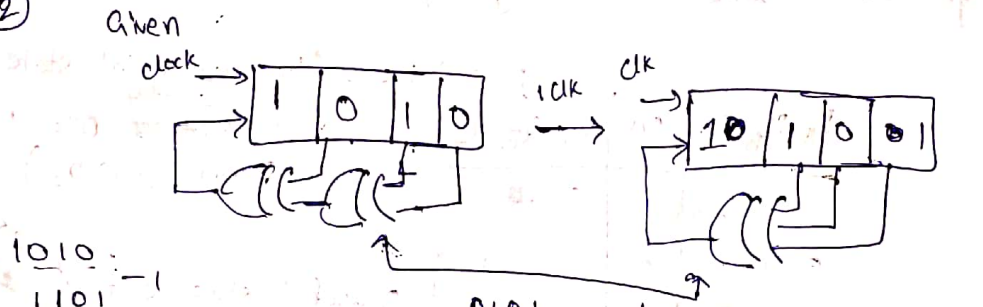
Eg: A 8 bit J.C is operated with 160 MHz clock

1) No of unused states is 240

2) fout is 10 MHz

## Problems

(2)




1010 – 1
1101
0110 – 2
0011 – 3
0001 – 4
1000 – 5
0100 – 6
1010 – 7

0101
1010

∴ 3 ~~clocks are~~ required

∴ 7 clocks

---

2014 2M

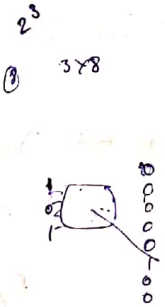
Q: Let $k=2^n$. A circuit is built by giving the o/p of n-bit binary counter as input to $n \times 2^n$ size decoder. This circuit is equivalent to

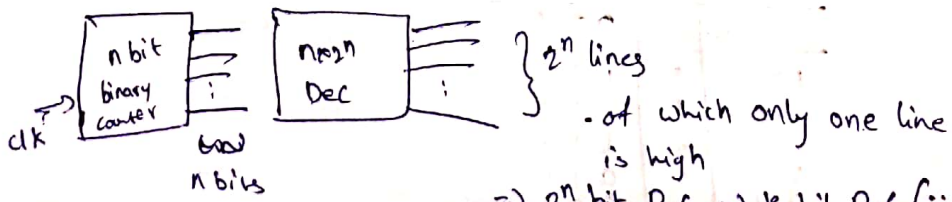
a) k bit binary upcounter

b) k bit binary down counter

c) k bit ring counter

d) k bit Johnson Counter

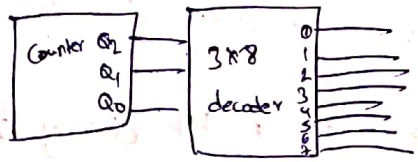





- of which only one line is high

⇒ $2^n$ bit R.C ⇒ k bit R.C ($\because 2^n = k$)

Let n=3 ⇒ k=8



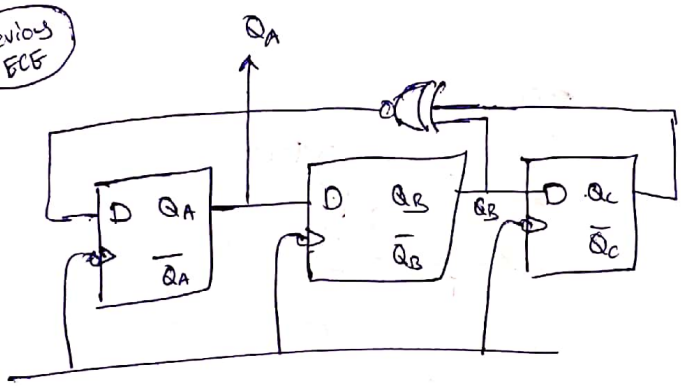

| $Q_2 Q_1 Q_0$ | $D_0 D_1 D_2 D_3 D_4 D_5 D_6 D_7$ |
|---|---|
| 0 0 0 | 1 0 0 0 0 0 0 0 |
| 0 0 1 | 0 1 0 0 0 0 0 0 |
| ⋮ | ⋮ |
| 1 1 1 | 0 0 0 0 0 0 0 1 |

∴ This represents 8 bit ~~Johnson~~ Ring Counter

i.e, k bit Ring Counter

## Problems

(8) Previous ECE




| $D_A D_B D_C$ | CLK | $Q_A$ $Q_B$ $Q_C$ |
|---|---|---|
| Initial | | 0 0 0 |
| 1 0 0 | ↑ | 1 0 0 |
| 1 1 0 | ↑ | 1 1 0 |
| 0 1 1 | ↑ | 0 1 1 |
| 1 0 1 | ↑ | 1 0 1 |
| 0 1 0 | ↑ | 0 1 0 |

$D_A = \cancel{Q_B Q_C + \bar{Q}_B \bar{Q}_C}$ $Q_B \odot Q_C$

$D_B = Q_A$

$D_C = Q_B$

$Q_A$ is 0 1 1 0 1 0 ....

## State table & State diagrams:

→ State table is also known as state transition table and it is ~~cal~~ closer to excitation table.

→ State diagram is the collection of circles. One circle represents one state, circle internal content is used to represent a state.

→ At the external side of the circle the numerator represents data i/p and denominator represents function o/p.

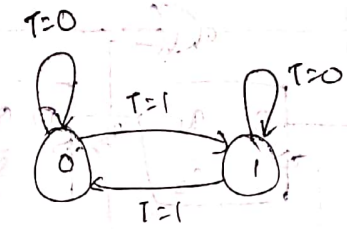
→ To draw the state diagram for n bit binary counter we require $2^n$ circles.

~~3 bit binary up counter state diagram:~~

State diagrams for Flip Flops:

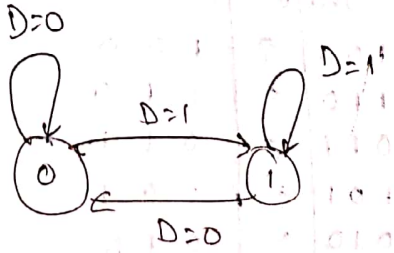
TFF

| Q | $Q_{t+1}$ | T |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



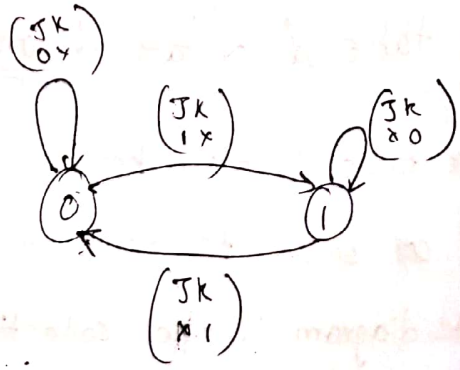

DFF

| $Q_t$ | $Q_{t+1}$ | D |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

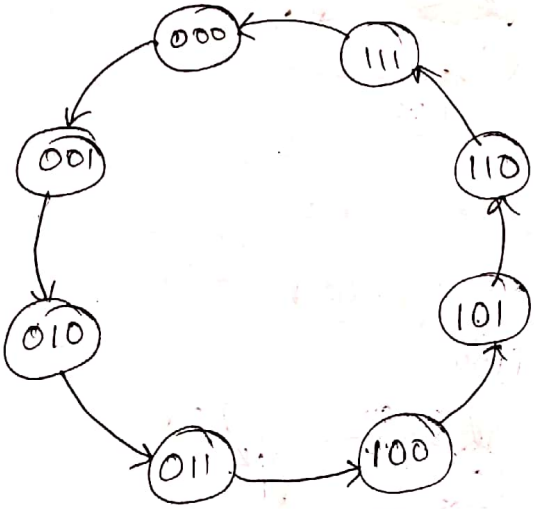



JKFF

| $Q_t$ | $Q_{t+1}$ | JK |
|---|---|---|
| 0 | 0 | 0X |
| 0 | 1 | 1X |
| 1 | 0 | X1 |
| 1 | 1 | X0 |



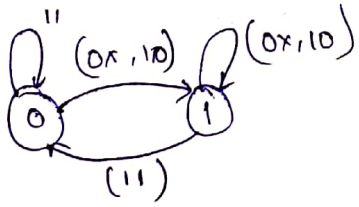
## 3 bit binary upCounter State diagram:




2015

Q: The following state diagram represents ____ gate

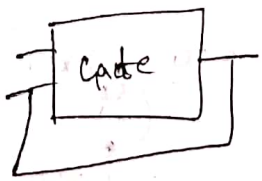
a) AND  b) OR  c) NAND  d) NOR




| A B | Y |
|---|---|
| 0 0 | 1 |
| 0 1 | 1 |
| 1 0 | 1 |
| 1 1 | 0 |

i.e, NAND gate

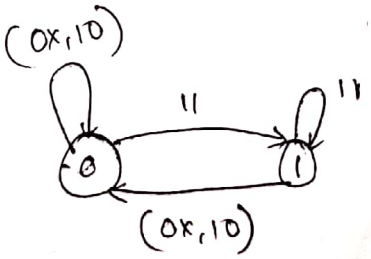
## State diagrams for logic gates:


Here Present I/P depends on previous o/p

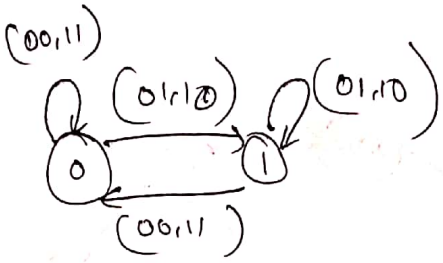
### AND gate

| $Q_t$ | $Q_{t+1}$ | AB |
|---|---|---|
| 0 | 0 | 0x,10 |
| 0 | 1 | 11 |
| 1 | 0 | 0x,10 |
| 1 | 1 | 11 |



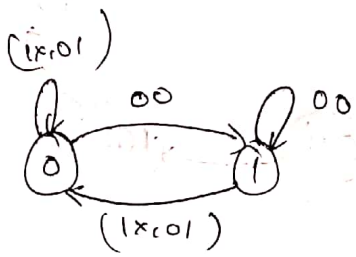
## XOR gate

| $Q_t$ | $Q_{t+1}$ | AB |
|---|---|---|
| 0 | 0 | 00, 11 |
| 0 | 1 | 01, 10 |
| 1 | 0 | 00, 11 |
| 1 | 1 | 01, 10 |



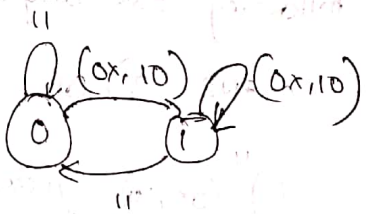

## NOR gate

| $Q_t$ | $Q_{t+1}$ | AB |
|---|---|---|
| 0 | 0 | 1X, 01 |
| 0 | 1 | 00 |
| 1 | 0 | 1X, 01 |
| 1 | 1 | 00 |



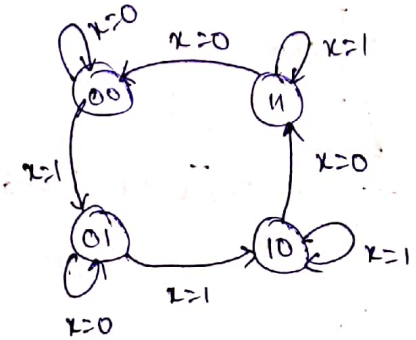

## NAND gate

| $Q_t$ | $Q_{t+1}$ | AB |
|---|---|---|
| 0 | 0 | 11 |
| 0 | 1 | (0x, 10) |
| 1 | 0 | 11 |
| 1 | 1 | (0x, 10) |




## Design the logic diagram for the below state diagram with TFF:

x = data i/p



| $Q_t$ PS | | $Q_{t+1}$ N.S | | | | |
|---|---|---|---|---|---|---|
| $Q_1 Q_0$ | x | $Q_1 Q_0$ | $T_1$ | $T_0$ | Y | |
| 0 0 | 0 | 0 0 | 0 | 0 | 0 | |
| 0 0 | 1 | 0 1 | 0 | 1 | 0 | |
| 0 1 | 0 | 0 1 | 0 | 0 | 0 | |
| 0 1 | 1 | 1 0 | 1 | 1 | 1 | 3 |
| 1 0 | 0 | 1 1 | 0 | 1 | 0 | |
| 1 0 | 1 | 1 0 | 0 | 0 | 1 | 5 |
| 1 1 | 0 | 0 0 | 1 | 1 | 1 | 6 |
| 1 1 | 1 | 0 1 | 0 | 0 | 1 | 7 |

Carry in FA

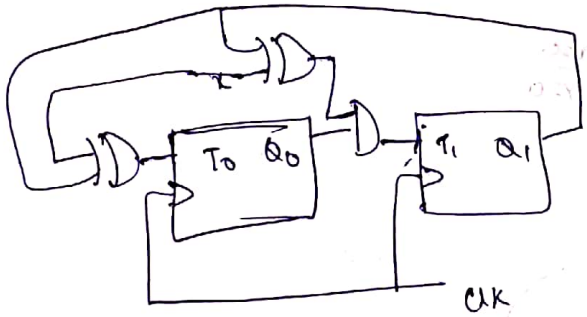
$T_1$

| $Q_1$ \ $Q_0 x$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | | |
| 1 | 1 | | | 1 |

$T_0 = \bar{Q}_1 Q_0 x + Q_1 Q_0 \bar{x}$

$= Q_0 (Q_1 \oplus x)$

Similarly $T_0 = Q_1 \oplus x$

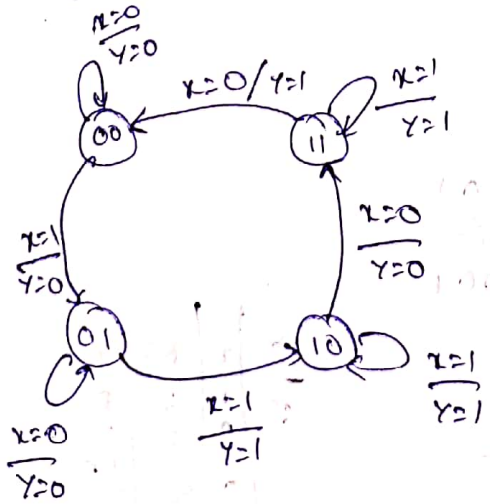



ckt 1

Now consider if we want generate o/p function (Y) for carry in FA

The O/p function Y is shown in above state table. The

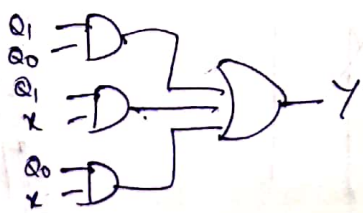
State diagram is




for carry in FA

$\Sigma m = (3, 5, 6, 7)$

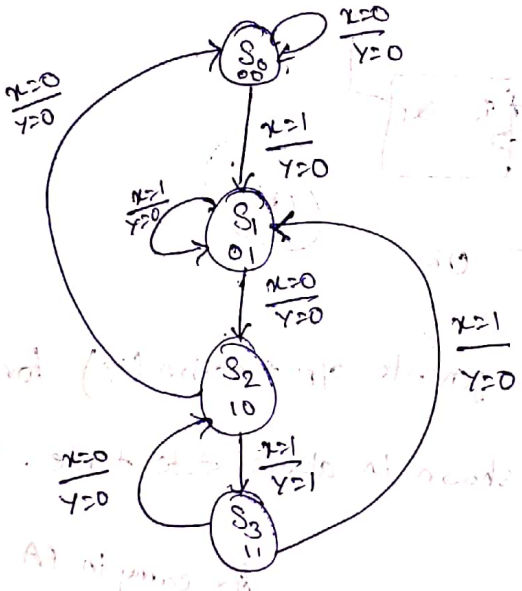
Also we add below ckt to above (ckt 1)



→ ~~The sequential~~

⇒ The sequential synchronous circuit is also known as Finite state Machine (FSM).

Q: The state diagram for the FSM is shown below. The initial state of the FSM is $S_0$. If the input data sequence is 1010110100110 1, starting with MSB the number of time output(Y) becomes high is 4




Initial state $S_0$

1 0 1 0 1 1 0 1 0 0 1 1 0 1

$S_1$ $S_2$ $S_3$ $S_2$ $S_3$ $S_1$ $S_2$ $S_3$ $S_2$ $S_0$ $S_1$ $S_1$ $S_2$ $S_3$

0 0 1 0 1 0 0 1 0 0 0 0 0 1

Method 2:

| PS | x | NS | Y |
|---|---|---|---|
| $S_0$ | 1 | | |
| | 0 | | |
| | 1 | | |
| | 0 | | |
| | 1 | | |
| | 1 | | |
| | 0 | | |
| | 1 | | |
| | 0 | | |
| | 0 | | |
| | 1 | | |
| | 1 | | |
| | 0 | | |
| | 1 | | |

| PS | x | NS | Y |
|---|---|---|---|
| $S_0$ | 1 | $S_1$ | 0 |
| $S_1$ | 0 | $S_2$ | 0 |
| $S_2$ | 1 | $S_3$ | 1 |
| $S_3$ | 0 | $S_2$ | 0 |
| $S_2$ | 1 | $S_3$ | 1 |
| $S_3$ | 1 | $S_1$ | 0 |
| $S_1$ | 0 | $S_2$ | 0 |
| $S_2$ | 1 | $S_3$ | 1 |
| $S_3$ | 0 | $S_2$ | 0 |
| $S_2$ | 0 | $S_0$ | 0 |
| $S_0$ | 1 | $S_2$ | 0 |
| $S_2$ | 1 | $S_1$ | 0 |
| $S_1$ | 0 | $S_2$ | 0 |
| $S_2$ | 1 | $S_3$ | 1 |

∴ 4 times